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OWL 2 Web Ontology Language RDF-Based Semantics

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Abstract

The OWL 2 Web Ontology Language, informally OWL 2, is an ontology language for the Semantic Web with formally defined meaning. OWL 2 ontologies provide classes, properties, individuals, and data values and are stored as Semantic Web documents. OWL 2 ontologies can be used along with information written in RDF, and OWL 2 ontologies themselves are primarily exchanged as RDF documents. The OWL 2 Document Overview describes the overall state of OWL 2, and should be read before other OWL 2 documents.
This document defines the RDF-compatible model-theoretic semantics of OWL 2.

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Summary of Changes

This Last Call Working Draft has undergone some small changes since the previous version of 21st April, 2009.

- The range of owl:predicate was adjusted to remove undesirable inferences.
- The RDF vocabulary for annotations was changed: owl:subject, owl:predicate and owl:object became, respectively, owl:annotatedSource, owl:annotatedProperty and owl:annotatedTarget.
- The name of rdf:text was changed to rdf:PlainLiteral.
- Some minor errors and infelicities were corrected.
- Some minor editorial changes were made.

Please Comment By 29 July 2009

The OWL Working Group seeks public feedback on this Working Draft. Please send your comments to public-owl-comments@w3.org (public archive). If possible, please offer specific changes to the text that would address your concern. You may also wish to check the Wiki Version of this document and see if the relevant text has already been updated.

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Editor's Note: Outstanding Editorial Work: Some editorial work has been deferred to the time right before publication as a Proposed Recommendation (PR), when the content of the document can be considered stable:

- The proof of the correspondence theorem (Section 7.3) might still need some further refinement.
- There are several item lists with both item bullets and numbers or letters. This will be changed into numbers/letters only after the proof of the correspondence theorem has been refined.
- Non-breakable whitespace will be put in formulae where appropriate.

1 Introduction (Informative)

This document defines the RDF-compatible model-theoretic semantics of OWL 2, referred to as the *"OWL 2 RDF-Based Semantics"*. The OWL 2 RDF-Based Semantics gives a formal meaning to every *RDF graph* [*RDF Concepts*] and is fully compatible with the *RDF Semantics specification* [*RDF Semantics*]. The specification provided here is the successor to the original *OWL 1 RDF-Compatible Semantics* specification [*OWL 1 RDF-Compatible Semantics*].

Technically, the OWL 2 RDF-Based Semantics is defined as a *semantic extension* of "D-Entailment" (RDFS with datatype support), as specified in [*RDF Semantics*]. In other words, the meaning given to an RDF graph by the OWL 2 RDF-Based Semantics includes the meaning given to the graph by the semantics of RDFS with datatypes, and additional meaning is given to all the language constructs of OWL 2, such as boolean connectives, sub property chains and qualified cardinality restrictions (see the OWL 2 Structural Specification [*OWL 2 Specification*] for further information on all the language constructs of OWL 2). The definition of the semantics for the extra constructs follows the same design principles that have been applied to the RDF Semantics.

The content of this document is not meant to be self-contained, but builds on top of the RDF Semantics document [*RDF Semantics*] by adding those aspects that are specific to OWL 2. Hence, the complete definition of the OWL 2 RDF-Based Semantics is given by the *combination* of both the RDF Semantics document and the document at hand. In particular, the terminology used in the RDF Semantics is reused here, except for cases where a conflict exists with the rest of the OWL 2 specification.

The following paragraphs outline the document's structure and content, and provide an overview of some of the distinguishing features of the OWL 2 RDF-Based Semantics.

According to Section 2, the *syntax* over which the OWL 2 RDF-Based Semantics is defined is the set of all *RDF graphs* [*RDF Concepts*]. Every such RDF graph is given a precise formal meaning by the OWL 2 RDF-Based Semantics. The language that is determined by RDF graphs being interpreted using the OWL 2 RDF-Based Semantics is called *"OWL 2 Full"*. In this document, RDF graphs are also called *"OWL 2 Full ontologies"*, or simply *"ontologies"*, unless there is any risk of confusion.

The OWL 2 RDF-Based Semantics interprets the RDF and RDFS *vocabularies* [*RDF Semantics*] and the *OWL 2 RDF-Based vocabulary*, together with an extended set of *datatypes* and their constraining *facets* (see Section 3).

OWL 2 RDF-Based interpretations (Section 4) are defined on a *universe* that is divided into *parts*, namely *individuals*, *classes*, and *properties*, which are identified with their RDF counterparts (see Figure 1). In particular, the part of individuals equals the whole universe. This means that all classes and properties are also individuals in their own right. Further, every name interpreted by an OWL 2 RDF-Based interpretation denotes an individual.

The three basic parts are further divided into subparts as follows. The part of individuals subsumes the part of *data values*, which comprises the denotations of all literals. Also subsumed by the individuals is the part of *ontologies*. The part of classes subsumes the part of *datatypes*, which are classes entirely consisting of data values. Finally, the part of properties subsumes the parts of *object properties*, *data properties*, *ontology properties* and *annotation properties*. In particular, the part of object properties equals the whole part of properties, and all other kinds of properties are therefore also object properties.

For *annotations properties* note that annotations cannot be considered "semantic-free" under the OWL 2 RDF-Based Semantics. Just like every other triple or set of triples occurring in an RDF graph, an annotation is assigned a truth value by any given OWL 2 RDF-Based interpretation. Hence, although annotations are meant to be "semantically weak", i.e. their formal meaning does not significantly exceed that coming from the RDF Semantics specification, adding an annotation may still change the meaning of an ontology. A similar discussion holds for statements that are built from *ontology properties*, such as `owl:imports`, which are used to define relationships between two ontologies.

Every class represents a specific set of individuals, called the *class extension* of the class, written as "ICEXT(*C*)". An individual *a* is an instance of a given class *C* exactly if *a* is a member of the class extension of *C*. Since a class is itself an individual under the OWL 2 RDF-Based Semantics, classes are distinguished from their respective class extensions. This distinction allows, for example, for a class to be an instance of itself by being a member of its own class extension. Also, two classes may be equivalent by sharing the same class extension, though still being

different individuals, i.e., they do not need to share the same properties. Similarly, every property has a *property extension*, written as "IEXT(*p*)", associated with it that consists of pairs of individuals. An individual a_1 has a relationship to another individual a_2 based on a given property *p*, exactly if the pair $\langle a_1 , a_2 \rangle$ is a member of the property extension of *p*. Again, properties are distinguished from their property extensions.

Individuals may play different roles. For example, an individual can be both a data property and an annotation property, since the different parts of the universe of an OWL 2 RDF-Based interpretation are not required to be mutually disjoint. Or an individual can be both a class and a property, since a class extension and a property extension may independently be associated with the same individual.

The main part of the OWL 2 RDF-Based Semantics is Section 5, which specifies a formal meaning for all the OWL 2 language constructs by means of the *OWL 2 RDF-Based semantic conditions*. These semantic conditions extend all the semantic conditions given in [*RDF Semantics*]. The OWL 2 RDF-Based semantic conditions effectively determine which sets of RDF triples are assigned a specific meaning, and what this meaning is. For example, there exist semantic conditions that allow to interpret the RDF triple "*C* `owl:disjointWith` *D*" to mean that the denotations of the IRIs *C* and *D* have disjoint class extensions.

There is usually no need to provide *localizing information* (e.g. by means of "typing triples") for the IRIs occurring in an ontology. As for the RDF Semantics, the OWL 2 RDF-Based semantic conditions have been designed to ensure that the denotation of any IRI will actually be in the appropriate part of the universe. For example, the RDF triple "*C* `owl:disjointWith` *D*" is sufficient to deduce that the denotations of the IRIs *C* and *D* are actually *classes*. It is not necessary to explicitly add additional typing triples "*C* `rdf:type rdfs:Class`" and "*D* `rdf:type rdfs:Class`" to the ontology.

In the RDF Semantics, this kind of "automatic localization" was to some extent achieved by so called *"axiomatic triples"* [*RDF Semantics*], such as "`rdfs:subClassOf rdf:type rdf:Property`" or "`rdfs:subClassOf rdfs:domain rdfs:Class`". However, there is no explicit collection of additional axiomatic triples for the OWL 2 RDF-Based Semantics but, instead, the specific axiomatic aspects of the OWL 2 RDF-Based Semantics are determined by a subset of the OWL 2 RDF-Based semantic conditions. Section 6 discusses axiomatic triples in general, and provides an example set of axiomatic triples that is compatible with the OWL 2 RDF-Based Semantics.

Section 7 compares the OWL 2 RDF-Based Semantics with the *OWL 2 Direct Semantics* [*OWL 2 Direct Semantics*]. While the OWL 2 RDF-Based Semantics is based on the RDF Semantics specification [*RDF Semantics*], the OWL 2 Direct Semantics is a *description logic* style semantics. Several fundamental differences exist between the two semantics, but there is also a strong relationship basically stating that the OWL 2 RDF-Based Semantics is able to reflect all logical conclusions of the OWL 2 Direct Semantics. This means that the OWL 2 Direct

Semantics can in a sense be regarded as a sub semantics of the OWL 2 RDF-Based Semantics. The precise relationship is given by the *OWL 2 correspondence theorem*.

Significant effort has been spent in keeping the design of the OWL 2 RDF-Based Semantics as close as possible to that of the original specification of the *OWL 1 RDF-Compatible Semantics* [*OWL 1 RDF-Compatible Semantics*]. The OWL 2 RDF-Based Semantics actually deviates from its predecessor in several aspects, in most cases due to serious technical problems that would have arisen from a conservative semantic extension. One important change is that, while there still exist so called *"comprehension conditions"* for the OWL 2 RDF-Based Semantics (see Section 8), these are *not* part of the normative set of semantic conditions anymore. The OWL 2 RDF-Based Semantics also corrects several errors of OWL 1. A list of differences between the two languages is given in Section 9.

The italicized keywords *must*, *must not*, *should*, *should not*, and *may* are used to specify normative features of OWL 2 documents and tools, and are interpreted as specified in RFC 2119 [*RFC 2119*].

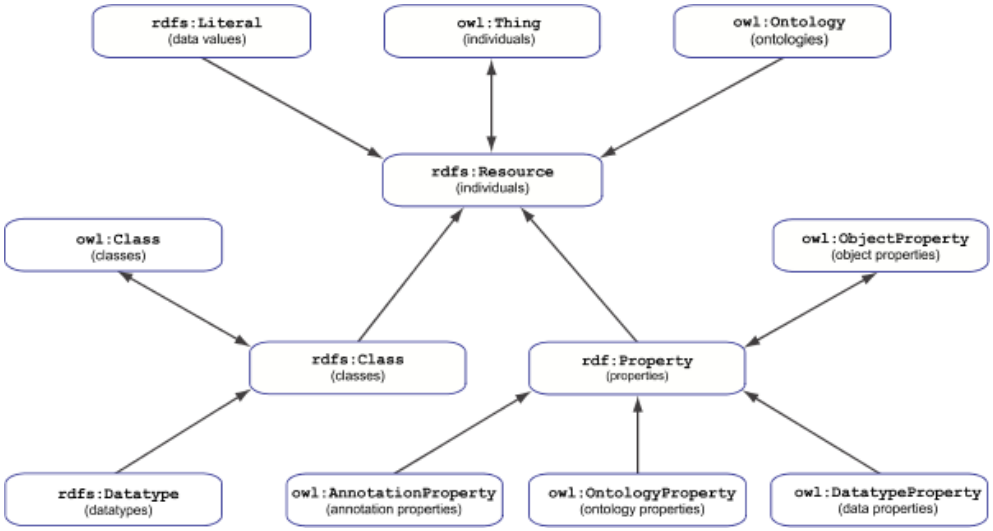

Figure 1: "Parts Hierarchy" of the OWL 2 RDF-Based Semantics

2 Ontologies

This section determines the *syntax* for the OWL 2 RDF-Based Semantics, and gives an overview on typical *content of ontologies* for ontology management tasks.

2.1 Syntax

Following Sections 0.2 and 0.3 of the RDF Semantics specification [*RDF Semantics*], the OWL 2 RDF-Based Semantics is defined on every ***RDF graph***

(Section 6.2 of [*RDF Concepts*]), i.e. on every set of ***RDF triples*** (Section 6.1 of [*RDF Concepts*]).

In accordance with the rest of the OWL 2 specification (see Section 2.3 of [*OWL 2 Specification*]), this document uses an extended notion of an RDF graph by allowing the RDF triples in an RDF graph to contain arbitrary ***IRIs*** ("Internationalized Resource Identifiers") according to [*RFC 3987*]. In contrast, the RDF Semantics specification [*RDF Semantics*] is defined on RDF graphs containing *URIs* [*RFC 2396*]. This change is backwards compatible with the RDF specification, since URIs are also IRIs.

Terminological note: The document at hand uses the term "IRI" in accordance with the rest of the OWL 2 specification (see Section 2.4 of [*OWL 2 Specification*]), whereas the RDF Semantics specification [*RDF Semantics*] uses the term "URI reference". According to [*RFC 3987*], the term "IRI" stands for an absolute resource identifier with optional fragment, which is what is being used throughout this document. In contrast, the term "IRI reference" additionally covers *relative* references, which are never used in this document.

Convention: In this document, IRIs are abbreviated in the way defined by Section 2.4 of [*OWL 2 Specification*], i.e., the abbreviations consist of a prefix name and a local part, such as "`prefix:localpart`".

The definition of an RDF triple according to Section 6.1 of [*RDF Concepts*] is restricted to cases where the *subject* of an RDF triple is an IRI or a *blank node* (Section 6.6 of [*RDF Concepts*]), and where the *predicate* of an RDF triple is an IRI. As a consequence, the definition does not treat cases, where, for example, the subject of a triple is a *literal* (Section 6.5 of [*RDF Concepts*]), as in `"s" ex:p ex:o`, or where the predicate of a triple is a blank node, as in `ex:s _:p ex:o`. In order to allow for interoperability with other existing and future technologies and tools, the document at hand does not explicitly forbid the use of ***generalized RDF graphs*** consisting of ***generalized RDF triples***, which are defined to allow for IRIs, literals and blank nodes to occur in the subject, predicate and object position. Thus, an RDF graph *may* contain generalized RDF triples, but an implementation is not required to support generalized RDF graphs. Note that every RDF graph consisting entirely of RDF triples according to Section 6.1 of [*RDF Concepts*] is also a generalized RDF graph.

Terminological notes: The term ***"OWL 2 Full"*** refers to the language that is determined by the set of all RDF graphs being interpreted using the OWL 2 RDF-Based Semantics. Further, in this document the term ***"OWL 2 Full ontology"*** (or simply ***"ontology"***, unless there is any risk of confusion) will be used interchangeably with the term "RDF graph".

2.2 Content of Ontologies (Informative)

While there do not exist any syntactic restrictions on the set of RDF graphs that can be interpreted by the OWL 2 RDF-Based Semantics, in practice an ontology will

often contain certain kinds of constructs that are aimed to support ontology management tasks. Examples are ***ontology headers*** and ***ontology IRIs***, as well as constructs that are about ***versioning***, ***importing*** and ***annotating*** of ontologies, including the concept of ***incompatibility*** between ontologies.

These topics are outside the scope of this semantics specification. Section 3 of [*OWL 2 Specification*] deals with these topics in detail, and can therefore be used as a guide on how to apply these constructs in OWL 2 Full ontologies accordingly. The mappings of all these constructs to their respective RDF encodings are defined in [*OWL 2 RDF Mapping*].

3 Vocabulary

This section specifies the *OWL 2 RDF-Based vocabulary*, and lists the names of the *datatypes* and *facets* used under the OWL 2 RDF-Based Semantics.

3.1 Standard Prefixes

Table 3.1 lists the standard prefix names and their prefix IRIs used in this document.

Table 3.1: Standard Prefixes

	Prefix Name	Prefix IRI
OWL	`owl`	`http://www.w3.org/2002/07/owl#`
RDF	`rdf`	`http://www.w3.org/1999/02/22-rdf-syntax-ns#`
RDFS	`rdfs`	`http://www.w3.org/2000/01/rdf-schema#`
XML Schema	`xsd`	`http://www.w3.org/2001/XMLSchema#`

3.2 Vocabulary Terms

Table 3.2 lists the IRIs of the *OWL 2 RDF-Based vocabulary*, which is the set of vocabulary terms that are specific for the OWL 2 RDF-Based Semantics. This vocabulary extends the RDF and RDFS vocabularies as specified by Sections 3.1 and 4.1 of [*RDF Semantics*], respectively. Table 3.2 excludes those IRIs that will be mentioned in Section 3.3 on datatype names or Section 3.4 on facet names.

Implementations are *not* required to support the IRI `owl:onProperties`, but *may* support it in order to realize *n-ary dataranges* with arity ≥ 2 (see Section 7 of [*OWL 2 Specification*] for further information).

Note: The use of the IRI `owl:DataRange` has been deprecated as of OWL 2. The IRI `rdfs:Datatype` *should* be used instead.

Table 3.2: OWL 2 RDF-Based vocabulary

```
owl:AllDifferent owl:AllDisjointClasses
owl:AllDisjointProperties owl:allValuesFrom
owl:annotatedProperty owl:annotatedSource
owl:annotatedTarget owl:Annotation owl:AnnotationProperty
owl:assertionProperty owl:AsymmetricProperty owl:Axiom
owl:backwardCompatibleWith owl:bottomDataProperty
owl:bottomObjectProperty owl:cardinality owl:Class
owl:complementOf owl:DataRange owl:datatypeComplementOf
owl:DatatypeProperty owl:deprecated owl:DeprecatedClass
owl:DeprecatedProperty owl:differentFrom
owl:disjointUnionOf owl:disjointWith owl:distinctMembers
owl:equivalentClass owl:equivalentProperty
owl:FunctionalProperty owl:hasKey owl:hasSelf owl:hasValue
owl:imports owl:incompatibleWith owl:intersectionOf
owl:InverseFunctionalProperty owl:inverseOf
owl:IrreflexiveProperty owl:maxCardinality
owl:maxQualifiedCardinality owl:members owl:minCardinality
owl:minQualifiedCardinality owl:NamedIndividual
owl:NegativePropertyAssertion owl:Nothing
owl:ObjectProperty owl:onClass owl:onDataRange
owl:onDatatype owl:oneOf owl:onProperty owl:onProperties
owl:Ontology owl:OntologyProperty owl:priorVersion
owl:propertyChainAxiom owl:propertyDisjointWith
owl:qualifiedCardinality owl:ReflexiveProperty
owl:Restriction owl:sameAs owl:someValuesFrom
owl:sourceIndividual owl:SymmetricProperty
owl:targetIndividual owl:targetValue owl:Thing
owl:topDataProperty owl:topObjectProperty
owl:TransitiveProperty owl:unionOf owl:versionInfo
owl:versionIRI owl:withRestrictions
```

3.3 Datatype Names

Table 3.3 lists the IRIs of the *datatypes* used in the OWL 2 RDF-Based Semantics. The datatype `rdf:XMLLiteral` is described in Section 3.1 of [*RDF Semantics*]. All other datatypes are described in Section 4 of [*OWL 2 Specification*]. The normative set of datatypes of the OWL 2 RDF-Based Semantics equals the set of datatypes described in Section 4 of [*OWL 2 Specification*].

Table 3.3: Datatypes of the OWL 2 RDF-Based Semantics

```
xsd:anyURI xsd:base64Binary xsd:boolean xsd:byte
xsd:dateTime xsd:dateTimeStamp xsd:decimal xsd:double
```

```
xsd:float xsd:hexBinary xsd:int xsd:integer xsd:language
xsd:long xsd:Name xsd:NCName xsd:negativeInteger
xsd:NMTOKEN xsd:nonNegativeInteger xsd:nonPositiveInteger
xsd:normalizedString rdf:PlainLiteral xsd:positiveInteger
owl:rational owl:real xsd:short xsd:string xsd:token
xsd:unsignedByte xsd:unsignedInt xsd:unsignedLong
xsd:unsignedShort rdf:XMLLiteral
```

Feature At Risk #1: *owl:rational* support

Note: This feature is "at risk" and may be removed from this specification based on feedback. Please send feedback to public-owl-comments@w3.org. For the current status see features "at risk" in OWL 2

The *owl:rational* datatype might be removed from OWL 2 if implementation experience reveals problems with supporting this datatype.

3.4 Facet Names

Table 3.4 lists the IRIs of the *facets* used in the OWL 2 RDF-Based Semantics. Each datatype listed in Section 3.3 has a (possibly empty) set of constraining facets. All facets are described in Section 4 of [*OWL 2 Specification*] in the context of their respective datatypes. The normative set of facets of the OWL 2 RDF-Based Semantics equals the set of facets described in Section 4 of [*OWL 2 Specification*].

In this specification, facets are used for defining *datatype restrictions* (see Section 5.7). For example, to refer to the set of all strings of length 5 one can restrict the datatype `xsd:string` (Section 3.3) by the facet `xsd:length` and the value 5.

Table 3.4: Datatype Facets of the OWL 2 RDF-Based Semantics

```
rdf:langRange xsd:length xsd:maxExclusive xsd:maxInclusive
xsd:maxLength xsd:minExclusive xsd:minInclusive
xsd:minLength xsd:pattern
```

4 Interpretations

The OWL 2 RDF-Based Semantics provides *vocabulary interpretations* and *vocabulary entailment* (see Section 2.1 of [*RDF Semantics*]) for the RDF and RDFS vocabularies and for the OWL 2 RDF-Based vocabulary. This section defines the concepts of an *OWL 2 RDF-Based datatype map* and an *OWL 2 RDF-Based interpretation*, and specifies what *satisfaction* of ontologies, *consistency* and *entailment* means under the OWL 2 RDF-Based Semantics. In addition, the so called *"parts" of the universe* of an OWL 2 RDF-Based interpretation are defined.

4.1 Datatype Maps

According to Section 5.1 of the RDF semantics specification [*RDF Semantics*], a ***datatype*** *d* has the following components:

- LS(*d*), the *lexical space* of *d*, which is a set of *lexical forms*;
- VS(*d*), the *value space* of *d*, which is a set of *data values*;
- L2V(*d*), the *lexical-to-value mapping* of *d*, which maps lexical forms in LS(*d*) to data values in VS(*d*).

Terminological notes: The document at hand uses the term *"data value"* in accordance with the rest of the OWL 2 specification (see Section 4 of [*OWL 2 Specification*]), whereas the RDF Semantics specification [*RDF Semantics*] uses the term *"datatype value"* instead. Further, the names "LS" and "VS", which stand for the lexical space and the value space of a datatype, respectively, are *not* used in the RDF Semantics specification, but have been introduced here for easier reference.

In this document, the basic definition of a datatype is extended to take *facets* into account. For information and an example on facets (see Section 3.4). Note that Section 5.1 of the RDF Semantics specification [*RDF Semantics*] explicitly permits that semantic extensions may impose more elaborate datatyping conditions than those listed above.

A ***datatype with facets*** *d* is a datatype that has the following additional components:

- FS(*d*), the *facet space* of *d*, which is a set of pairs of the form $\langle F , v \rangle$, where *F* is an IRI called the *constraining facet* and *v* is an arbitrary data value called the *constraining value*;
- F2V(*d*), the *facet-to-value mapping* of *d*, which maps each facet-value pair $\langle F , v \rangle$ in FS(*d*) to a subset of VS(*d*).

Note that it is not further specified what the nature of a facet IRI's denotation is, i.e. it is only known that a facet IRI denotes some individual. Semantic extensions *may* impose further restrictions on the denotations of facets. In fact, Section 5.3 will define additional restrictions on facets.

Also note that for a datatype *d* and a facet-value pair $\langle F , v \rangle$ in FS(*d*) the value *v* is not required to be included in the value space VS(*d*) of *d* itself. For example, the datatype `xsd:string` (Section 3.3) has the facet `xsd:length` (Section 3.4), which takes non-negative integers as its values, rather than strings.

In this document, it will always be assumed from now on that every datatype *d* is a datatype with facets. If the facet space FS(*d*) of a datatype *d* has not been explicitly defined, or if it is not derived from another datatype's facet space according to some well defined condition, then FS(*d*) is the empty set. Unless there is any risk of confusion, the term *"datatype"* will always refer to a datatype with facets.

Section 5.1 of the RDF Semantics specification [*RDF Semantics*] further defines a ***datatype map*** *D* to be a set of name-datatype pairs ⟨ *u* , *d* ⟩ consisting of an IRI *u* and a datatype *d*, such that no IRI appears twice in the set. As a consequence of what has said before, in this document every datatype map *D* will entirely consist of datatypes with facets.

The following definition specifies what an *OWL 2 RDF-Based datatype map* is.

Definition 4.1 (OWL 2 RDF-Based Datatype Map): A datatype map *D* is an *OWL 2 RDF-Based datatype map*, if and only if for every datatype name *u* listed in Section 3.3 and its respective set of constraining facets (Section 3.4) there is a name-datatype pair ⟨ *u*, *d* ⟩ in *D* with the specified lexical space LS(*d*), value space VS(*d*), lexical-to-value mapping L2V(*d*), facet space FS(*d*) and facet-to-value mapping F2V(*d*).

Note that Definition 4.1 does not prevent *additional* datatypes to be in an OWL 2 RDF-Based datatype map. For the special case of an OWL 2 RDF-Based datatype map *D* that exclusively contains the datatypes listed in Section 3.3, it is ensured that there are datatypes available for all the facet values, i.e., for every name-datatype pair ⟨ *u* , *d* ⟩ in *D* and for every facet-value pair ⟨ *F* , *v* ⟩ in FS(*d*) there exists a name-datatype pair ⟨ u^* , d^* ⟩ in *D* such that *v* is in VS(d^*).

4.2 Vocabulary Interpretations

From the RDF Semantics specification [*RDF Semantics*], let *V* be a set of literals and IRIs containing the RDF and RDFS vocabularies, and let *D* be a datatype map according to Section 5.1 of [*RDF Semantics*] (and accordingly Section 4.1). A ***D-interpretation*** *I* of *V* with respect to *D* is a tuple

I = ⟨ IR , IP , IEXT , IS , IL , LV ⟩ .

IR is the *universe* of *I*, i.e., a nonempty set that contains at least the denotations of literals and IRIs in *V*. IP is a subset of IR, the *properties* of *I*. LV, the *data values* of *I*, is a subset of IR that contains at least the set of plain literals (see Section 6.5 of [*RDF Concepts*]), and the value spaces of each datatype of *D*. IEXT is used to associate properties with their *property extension*, and is a mapping from IP to the powerset of IR × IR. IS is a mapping from *IRIs* in *V* to their denotations in IR. In particular, IS(*u*) = *d* for any name-datatype pair ⟨ *u* , *d* ⟩ in *D*. IL is a mapping from *typed literals* "*s*"^^*u* in *V* to their denotations in IR, where IL("*s*"^^*u*) = L2V(*d*)(*s*), provided that *d* is a datatype of *D*, IS(*u*) = *d*, and *s* is in the lexical space LS(*d*); otherwise IL("*s*"^^*u*) is not in LV.

Convention: Following the practice, as also introduced in Section 1.4 of [*RDF Semantics*], for a given interpretation *I* of a vocabulary *V* the notation "*I*(*x*)" will be used to denote "IL(*x*)" and "IS(*x*)" for the typed literals and IRIs *x* in *V*, respectively.

As detailed in [*RDF Semantics*], a D-interpretation has to meet all the semantic conditions for ground graphs and blank nodes, those for RDF interpretations and RDFS interpretations, and the "general semantic conditions for datatypes".

In this document, the basic definition of a D-interpretation is extended to take *facets* into account.

A ***D-interpretation with facets*** *I* is a D-interpretation for a datatype map *D* consisting entirely of datatypes with facets (Section 4.1), where *I* meets the following additional semantic conditions: for each name-datatype pair $\langle u , d \rangle$ in *D* and each facet-value pair $\langle F , v \rangle$ in the facet space FS(*d*)

- *F* is in the vocabulary *V* of *I*;
- a name-datatype pair $\langle u^* , d^* \rangle$ exists in *D*, such that *v* is in the value space VS(d^*).

In this document, it will always be assumed from now on that every D-interpretation *I* is a D-interpretation with facets. Unless there is any risk of confusion, the term *"D-interpretation"* will always refer to a D-interpretation with facets.

The following definition specifies what an *OWL 2 RDF-Based interpretation* is.

Definition 4.2 (OWL 2 RDF-Based Interpretation): Let *D* be an OWL 2 RDF-Based datatype map, and let *V* be a vocabulary that includes the RDF and RDFS vocabularies and the OWL 2 RDF-Based vocabulary together with all the datatype and facet names listed in Section 3. An *OWL 2 RDF-Based interpretation*, $I = \langle$ IR , IP , IEXT , IS , IL , LV $\rangle$, of *V* with respect to *D* is a D-interpretation of *V* with respect to *D* that meets all the extra semantic conditions given in Section 5.

4.3 Satisfaction, Consistency and Entailment

The following definitions specify what it means for an RDF graph to be *satisfied* by a given OWL 2 RDF-Based interpretation, to be *consistent* under the OWL 2 RDF-Based Semantics, and to *entail* another RDF graph.

The notion of *satisfaction* under the OWL 2 RDF-Based Semantics is based on the notion of satisfaction for D-interpretations and Simple interpretations, as defined in [*RDF Semantics*]. In essence, in order to satisfy an RDF graph, an interpretation *I* has to satisfy all the triples in the graph, i.e., for a triple of the form "*s p o*" it is necessary that the relationship $\langle I(s) , I(o) \rangle \in$ IEXT($I(p)$) holds (special treatment exists for blank nodes, as detailed in Section 1.5 of [*RDF Semantics*]). In other words, the given graph has to be compatible with the specific form of the IEXT mapping of *I*. The distinguishing aspect of *OWL 2 RDF-Based satisfaction* is that an interpretation *I* needs to meet all the OWL 2 RDF-Based semantic conditions (see Section 5), which have the effect of constraining the possible forms an IEXT mapping can have.

Definition 4.3 (OWL 2 RDF-Based Satisfaction): Let *G* be an RDF graph, let *D* be an OWL 2 RDF-Based datatype map, let *V* be a vocabulary that includes the RDF and RDFS vocabularies and the OWL 2 RDF-Based vocabulary together with all the datatype and facet names listed in Section 3, and let *I* be a D-interpretation of *V* with respect to *D*. *I OWL 2 RDF-Based satisfies G* with respect to *V* and *D* if and only if *I* is an OWL 2 RDF-Based interpretation of *V* with respect to *D* that satisfies *G* as a D-interpretation of *V* with respect to *D* according to [*RDF Semantics*].

Definition 4.4 (OWL 2 RDF-Based Consistency): Let *S* be a collection of RDF graphs, and let *D* be an OWL 2 RDF-Based datatype map. *S* is *OWL 2 RDF-Based consistent* with respect to *D* if and only if there is some OWL 2 RDF-Based interpretation *I* with respect to *D* of some vocabulary *V* that includes the RDF and RDFS vocabularies and the OWL 2 RDF-Based vocabulary together with all the datatype and facet names listed in Section 3), such that *I* OWL 2 RDF-Based satisfies all the RDF graphs in *S* with respect to *V* and *D*.

Definition 4.5 (OWL 2 RDF-Based Entailment): Let S_1 and S_2 be collections of RDF graphs, and let *D* be an OWL 2 RDF-Based datatype map. S_1 *OWL 2 RDF-Based entails* S_2 with respect to *D* if and only if for every OWL 2 RDF-Based interpretation *I* with respect to *D* of any vocabulary *V* that includes the RDF and RDFS vocabularies and the OWL 2 RDF-Based vocabulary together with all the datatype and facet names listed in Section 3 the following holds: If *I* OWL 2 RDF-Based satisfies all the RDF graphs in S_1 with respect to *V* and *D*, then *I* OWL 2 RDF-Based satisfies all the RDF graphs in S_2 with respect to *V* and *D*.

4.4 Parts of the Universe

Table 4.1 defines the *"parts"* of the universe of a given OWL 2 RDF-Based interpretation *I*.

The second column tells the *name* of the part. The third column gives a *definition* of the part in terms of the mapping IEXT of *I*, and by referring to particular terms of the RDF, RDFS and OWL 2 RDF-Based vocabularies.

As an example, the part of all datatypes is named "IDC", and it is defined as the set of all individuals *x* for which the relationship "$\langle x , I(\texttt{rdfs:Datatype}) \rangle \in$ IEXT(I(`rdf:type`))" holds. According to the semantics of `rdf:type`, as defined in Section 4.1 of [*RDF Semantics*], this means that the name "IDC" denotes the class extension (see Section 4.5) of I(`rdfs:Datatype`).

Table 4.1: Parts of the Universe

	Name of Part *S*	**Definition of *S* as $\{ x \in IR \mid \langle x , I(E) \rangle \in$ IEXT(I(`rdf:type`)) $\}$ where IRI *E* is**
individuals	IR	`rdfs:Resource`

data values	LV	`rdfs:Literal`
ontologies	IX	`owl:Ontology`
classes	IC	`rdfs:Class`
datatypes	IDC	`rdfs:Datatype`
properties	IP	`rdf:Property`
data properties	IODP	`owl:DatatypeProperty`
ontology properties	IOXP	`owl:OntologyProperty`
annotation properties	IOAP	`owl:AnnotationProperty`

4.5 Class Extensions

The mapping ICEXT from IC to the powerset of IR, which associates classes with their *class extension*, is defined for every $c \in$ IC as

$$\text{ICEXT}(c) = \{ x \in \text{IR} \mid \langle x , c \rangle \in \text{IEXT}(I(\texttt{rdf:type})) \} .$$

5 Semantic Conditions

This section defines the semantic conditions of the OWL 2 RDF-Based Semantics. The semantic conditions presented here are basically only those for the specific constructs of OWL 2. The complete set of semantic conditions for the OWL 2 RDF-Based Semantics is the combination of the semantic conditions presented here and the semantic conditions for Simple Entailment, RDF, RDFS and D-Entailment, as specified in the RDF Semantics specification [*RDF Semantics*].

Section 5.1 specifies semantic conditions for the different parts of the universe (as defined in Section 4.4) of the OWL 2 RDF-Based interpretation being considered. Section 5.2 and Section 5.3 list semantic conditions for the classes and the properties of the OWL 2 RDF-Based vocabulary. In the rest of this section, the OWL 2 RDF-Based semantic conditions for the different language constructs of OWL 2 are specified.

Conventions used in this Section

iff: Throughout this section the term "iff" is used as a shortform for "if and only if".

Conjunctive commas: A comma (", ") separating two assertions in a semantic condition, as in "$c \in$ IC , $p \in$ IP", is read as a logical *"and"*. Further, a comma separating two variables, as in "c, $d \in$ IC", is used for abbreviating two comma separated assertions, "$c \in$ IC , $d \in$ IC" in this example.

Unscoped variables: If no explicit scope is given for a variable "*x*", as in "∀ *x* : …" or "{ *x* | … }", then "*x*" is unconstrained, which means *x* ∈ IR, i.e. "*x*" denotes an arbitrary individual in the universe.

Set cardinality: For a set *S*, an expression of the form "#*S*" means the number of elements in *S*.

Sequence expressions: An expression of the form "*s* sequence of a_1 , … , $a_n \in S$" means that "*s*" represents a list of $n \geq 0$ individuals a_1 , … , a_n, all of them being members of the set *S*. Precisely, $s = I(\texttt{rdf:nil})$ for $n = 0$; and for $n > 0$ there exist $z_1 \in$ IR , … , $z_n \in$ IR, such that

$s = z_1$,
$a_1 \in S$, $\langle z_1 , a_1 \rangle \in$ IEXT($I(\texttt{rdf:first})$) , $\langle z_1 , z_2 \rangle \in$ IEXT($I(\texttt{rdf:rest})$) ,
… ,
$a_n \in S$, $\langle z_n , a_n \rangle \in$ IEXT($I(\texttt{rdf:first})$) , $\langle z_n , I(\texttt{rdf:nil}) \rangle \in$ IEXT($I(\texttt{rdf:rest})$) .

Note, as mentioned in Section 3.3.3 of [*RDF Semantics*], there are no semantic constraints that enforce "well-formed" sequence structures. So, for example, it is possible for a sequence head *s* to refer to more than one sequence.

Set names: The following names are used as convenient abbreviations for certain sets:

- ISEQ: The set of all sequences. This set equals the class extension of `rdf:List`, i.e., ISEQ := ICEXT($I(\texttt{rdf:List})$).
- INNI: The set of all non-negative integers. This set equals the value space of the datatype `xsd:nonNegativeInteger`, i.e., INNI := ICEXT($I(\texttt{xsd:nonNegativeInteger})$), but is also subsumed by the value spaces of other numerical datatypes, such as `xsd:integer`.

Notes on the Form of Semantic Conditions (Informative)

One design goal of OWL 2 was to ensure an appropriate degree of alignment between the OWL 2 RDF-Based Semantics and the OWL 2 Direct Semantics [*OWL 2 Direct Semantics*] under the different constraints the two semantics have to meet. The way this semantic alignment is described is via the *OWL 2 Correspondence Theorem* in Section 7.2. For this theorem to hold, the semantic conditions that treat the RDF encodings of OWL 2 *axioms* (compare Section 3.2.5 of [*OWL 2 RDF Mapping*] and Section 9 of [*OWL 2 Specification*]), such as inverse property axioms, must have the form of "iff" ("if-and-only-if") conditions. This means that these semantic conditions completely determine the semantics of these construct encodings. On the other hand, the RDF encodings of OWL 2 *expressions* (compare Section 3.2.4 of [*OWL 2 RDF Mapping*] and Sections 6 – 8 of [*OWL 2 Specification*]), such as property restrictions, are treated by "if-then" conditions. These weaker semantic conditions for expressions are sufficient for the

correspondence theorem to hold, so there is no necessity to define stronger "iff" conditions under the OWL 2 RDF-Based Semantics for these language constructs.

Special cases are the semantic conditions for boolean connectives of classes and enumerations of individuals. These language constructs build OWL 2 expressions. But for backwards compatibility reasons there are also RDF encodings of *axioms* based on the vocabulary for these language constructs (see Table 18 in Section 3.2.5 of [*OWL 2 RDF Mapping*]). For example, an RDF expression of the form

```
ex:c1 owl:unionOf ( ex:c2 ex:c3 ) .
```

is mapped by the reverse RDF mapping to an OWL 2 axiom that states the equivalence of the class denoted by `ex:c1` with the union of the classes denoted by `ex:c2` and `ex:c3`. In order to ensure that the correspondence theorem holds, and in accordance with the original OWL 1 RDF-Compatible Semantics specification [*OWL 1 RDF-Compatible Semantics*], the semantic conditions for the mentioned language constructs are therefore "iff" conditions.

Further, special treatment exists for OWL 2 axioms that have *multi-triple representations* in RDF, where the different triples share a common *"root node"*, such as the blank node "`_:x`" in the following example:

```
_:x rdf:type owl:AllDisjointClasses .
_:x owl:members ( ex:c1 ex:c2 ) .
```

In essence, the semantic conditions for the encodings of these language constructs are "iff" conditions, as usual for axioms. However, in order to cope with the specific syntactic aspect of a "root node", the "iff" conditions of these language constructs have been split into two "if-then" conditions, where the "if-then" condition representing the right-to-left direction contains an additional premise of the form "$\exists$ *z* $\in$ IR". The purpose of this premise is to ensure the existence of an individual that is needed to satisfy the root node under the OWL 2 RDF-Based semantics. The language constructs in question are *n-ary disjointness axioms* in Section 5.10, and *negative property assertions* in Section 5.15.

The "if-then" semantic conditions in this section sometimes do not explicitly list all typing statements in their consequent that one might expect. For example, the semantic condition for `owl:someValuesFrom` restrictions in Section 5.6 does not list the statement "*x* $\in$ ICEXT(*I*(`owl:Restriction`))" on its right hand side. Consequences are generally not mentioned, if they can already be deduced by other means. Often, these redundant consequences follow from the semantic conditions for *classes* and *properties* in Section 5.2 and Section 5.3, respectively, occasionally in connection with the semantic conditions for the *parts of the universe* in Section 5.1. In the example above, the omitted consequence can be obtained from the third column of the entry for `owl:someValuesFrom` in the table in Section 5.3, which determines that IEXT(*I*(`owl:someValuesFrom`)) $\subseteq$ ICEXT(*I*(`owl:Restriction`)) × IC.

5.1 Semantic Conditions for the Parts of the Universe

Table 5.1 lists the semantic conditions for the parts of the universe of the OWL 2 RDF-Based interpretation being considered. Additional semantic conditions affecting the parts are given in Section 5.2.

The first column tells the *name* of the part, as defined in Section 4.4. The second column defines certain *conditions* on the part. In most cases, the column specifies for the part by which other part it is subsumed, and thus the position of the part in the "parts hierarchy" of the universe is narrowed down. The third column provides further *information about the instances* of those parts that consist of classes or properties. In general, if the part consists of classes, then for the class extensions of the member classes it is specified by which part of the universe they are subsumed. If the part consists of properties, then the domains and ranges of the member properties are determined.

Table 5.1: Semantic Conditions for the Parts of the Universe

Name of Part *S*	Conditions on *S*	Conditions on Instances *x* of *S*
IR	$S \neq \emptyset$	
LV	$S \subseteq$ IR	
IX	$S \subseteq$ IR	
IC	$S \subseteq$ IR	ICEXT(x) $\subseteq$ IR
IDC	$S \subseteq$ IC	ICEXT(x) $\subseteq$ LV
IP	$S \subseteq$ IR	IEXT(x) $\subseteq$ IR $\times$ IR
IODP	$S \subseteq$ IP	IEXT(x) $\subseteq$ IR $\times$ LV
IOXP	$S \subseteq$ IP	IEXT(x) $\subseteq$ IX $\times$ IX
IOAP	$S \subseteq$ IP	IEXT(x) $\subseteq$ IR $\times$ IR

5.2 Semantic Conditions for the Vocabulary Classes

Table 5.2 lists the semantic conditions for the classes that have IRIs in the OWL 2 RDF-Based vocabulary. In addition, the table contains all those classes with IRIs in the RDF and RDFS vocabularies that represent parts of the universe of the OWL 2 RDF-Based interpretation being considered (Section 4.4). The semantic conditions for the remaining classes with names in the RDF and RDFS vocabularies can be found in the RDF Semantics specification [*RDF Semantics*].

The first column tells the *name* of the class. The second column defines of what particular *kind* a class is, i.e. whether it is a general class (a member of the part IC) or a datatype (a member of IDC). The third column specifies for the class extension of the class by which part of the universe (Section 4.4) it is *subsumed*: from an entry of the form "ICEXT($I(C)$) $\subseteq$ S", for a class name C and a set S, and given an RDF triple of the form "u `rdf:type` C", one can deduce that the relationship "$I(u) \in S$" holds. Note that some entries are of the form "ICEXT($I(C)$) = S", which means that the class extension is exactly specified to be that set. See Section 5.1 for further semantic conditions on those classes that represent *parts*.

Not included in this table are the *datatypes* of the OWL 2 RDF-Based Semantics with IRIs listed in Section 3.3. For each such datatype IRI E, the following semantic conditions hold (as a consequence of the fact that E is a member of the datatype map of every OWL 2 RDF-Based interpretation according to Definition 4.2, and by the "General semantic conditions for datatypes" listed in Section 5.1 of [*RDF Semantics*]):

- $I(E) \in$ IDC
- ICEXT($I(E)$) $\subseteq$ LV

Table 5.2: Semantic Conditions for the Vocabulary Classes

IRI E	$I(E)$	ICEXT($I(E)$)
`owl:AllDifferent`	$\in$ IC	$\subseteq$ IR
`owl:AllDisjointClasses`	$\in$ IC	$\subseteq$ IR
`owl:AllDisjointProperties`	$\in$ IC	$\subseteq$ IR
`owl:Annotation`	$\in$ IC	$\subseteq$ IR
`owl:AnnotationProperty`	$\in$ IC	= IOAP
`owl:AsymmetricProperty`	$\in$ IC	$\subseteq$ IP
`owl:Axiom`	$\in$ IC	$\subseteq$ IR
`rdfs:Class`	$\in$ IC	= IC
`owl:Class`	$\in$ IC	= IC
`owl:DataRange`	$\in$ IC	= IDC
`rdfs:Datatype`	$\in$ IC	= IDC
`owl:DatatypeProperty`	$\in$ IC	= IODP
`owl:DeprecatedClass`	$\in$ IC	$\subseteq$ IC
`owl:DeprecatedProperty`	$\in$ IC	$\subseteq$ IP

owl:FunctionalProperty	∈ IC	⊆ IP
owl:InverseFunctionalProperty	∈ IC	⊆ IP
owl:IrreflexiveProperty	∈ IC	⊆ IP
rdfs:Literal	∈ IDC	= LV
owl:NamedIndividual	∈ IC	⊆ IR
owl:NegativePropertyAssertion	∈ IC	⊆ IR
owl:Nothing	∈ IC	= ∅
owl:ObjectProperty	∈ IC	= IP
owl:Ontology	∈ IC	= IX
owl:OntologyProperty	∈ IC	= IOXP
rdf:Property	∈ IC	= IP
owl:ReflexiveProperty	∈ IC	⊆ IP
rdfs:Resource	∈ IC	= IR
owl:Restriction	∈ IC	⊆ IC
owl:SymmetricProperty	∈ IC	⊆ IP
owl:Thing	∈ IC	= IR
owl:TransitiveProperty	∈ IC	⊆ IP

5.3 Semantic Conditions for the Vocabulary Properties

Table 5.3 lists the semantic conditions for the properties that have IRIs in the OWL 2 RDF-Based vocabulary. In addition, the table contains all those properties with IRIs in the RDFS vocabulary that are specified to be annotation properties under the OWL 2 RDF-Based Semantics. The semantic conditions for the remaining properties with names in the RDFS vocabulary can be found in the RDF Semantics specification [*RDF Semantics*].

The first column tells the *name* of the property. The second column defines of what particular *kind* a property is, i.e. whether it is a general property (a member of the part IP), a datatype property (a member of IODP), an ontology property (a member of IOXP) or an annotation property (a member of IOAP). The third column specifies the *domain and range* of the property: from an entry of the form

"IEXT($I(p)$) ⊆ S_1 × S_2", for a property name p and sets S_1 and S_2, and given an RDF triple of the form "s p o", one can deduce that the relationships "$I(s) \in S_1$" and "$I(o) \in S_2$" hold. Note that some entries are of the form "IEXT($I(p)$) = S_1 × S_2", which means that the property extension is exactly specified to be the Cartesian product of the two sets.

Not included in this table are the *datatype facets* of the OWL 2 RDF-Based Semantics with IRIs listed in Section 3.4, which are used to specify datatype restrictions (see Section 5.7). For each such datatype facet IRI E, the following semantic conditions *extend* the basic semantics specification that has been given for *datatypes with facets* in Section 4.1:

- $I(E) \in$ IP
- IEXT($I(E)$) ⊆ IR × LV

Implementations are *not* required to support the semantic condition for `owl:onProperties`, but *may* support it in order to realize *n-ary dataranges* with arity ≥ 2 (see Section 7 of [*OWL 2 Specification*] for further information).

Informative notes:

`owl:topObjectProperty` relates every two individuals in the universe with each other. Likewise, `owl:topDataProperty` relates every individual with every data value. Further, `owl:bottomObjectProperty` and `owl:bottomDataProperty` stand both for the *empty* relationship.

The ranges of the properties `owl:deprecated` and `owl:hasSelf` are not restricted in any form, and, in particular, they are not restricted to be boolean values. The actual object values of these properties do not have any intended meaning, but could as well have been defined to be of any other value. Therefore, the semantics given here are of a form that the values can be arbitrarily chosen without leading to any non-trivial semantic conclusions. It is, however, recommended to still use an object literal of the form `"true"^^xsd:boolean` in ontologies, in order to not get in conflict with the required usage of these properties in scenarios that ask for applying the reverse RDF mapping (compare Table 13 in Section 3.2.4 of [*OWL 2 RDF Mapping*] for `owl:hasSelf`, and Section 5.5 of [*OWL 2 Specification*] for `owl:deprecated`).

The range of the property `owl:annotatedProperty` is unrestricted in order to avoid undesired semantic side effects from an annotation, when the annotated axiom or annotation is not contained in the ontology.

Table 5.3: Semantic Conditions for the Vocabulary Properties

IRI E	$I(E)$	IEXT($I(E)$)
`owl:allValuesFrom`	∈ IP	⊆ ICEXT(I(`owl:Restriction`)) × IC
`owl:annotatedProperty`	∈ IP	⊆ IR × IR

owl:annotatedSource	∈ IP	⊆ IR × IR
owl:annotatedTarget	∈ IP	⊆ IR × IR
owl:assertionProperty	∈ IP	⊆ ICEXT(*I*(owl:NegativePropertyAssertion)) × IP
owl:backwardCompatibleWith	∈ IOXP	⊆ IX × IX
owl:bottomDataProperty	∈ IODP	= ∅
owl:bottomObjectProperty	∈ IP	= ∅
owl:cardinality	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × INNI
rdfs:comment	∈ IOAP	⊆ IR × LV
owl:complementOf	∈ IP	⊆ IC × IC
owl:datatypeComplementOf	∈ IP	⊆ IDC × IDC
owl:deprecated	∈ IOAP	⊆ IR × IR
owl:differentFrom	∈ IP	⊆ IR × IR
owl:disjointUnionOf	∈ IP	⊆ IC × ISEQ
owl:disjointWith	∈ IP	⊆ IC × IC
owl:distinctMembers	∈ IP	⊆ ICEXT(*I*(owl:AllDifferent)) × ISEQ
owl:equivalentClass	∈ IP	⊆ IC × IC
owl:equivalentProperty	∈ IP	⊆ IP × IP
owl:hasKey	∈ IP	⊆ IC × ISEQ
owl:hasSelf	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × IR
owl:hasValue	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × IR
owl:imports	∈ IOXP	⊆ IX × IX

owl:incompatibleWith	∈ IOXP	⊆ IX × IX
owl:intersectionOf	∈ IP	⊆ IC × ISEQ
owl:inverseOf	∈ IP	⊆ IP × IP
rdfs:isDefinedBy	∈ IOAP	⊆ IR × IR
rdfs:label	∈ IOAP	⊆ IR × LV
owl:maxCardinality	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × INNI
owl:maxQualifiedCardinality	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × INNI
owl:members	∈ IP	⊆ IR × ISEQ
owl:minCardinality	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × INNI
owl:minQualifiedCardinality	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × INNI
owl:onClass	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × IC
owl:onDataRange	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × IDC
owl:onDatatype	∈ IP	⊆ IDC × IDC
owl:oneOf	∈ IP	⊆ IC × ISEQ
owl:onProperty	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × IP
owl:onProperties	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × ISEQ
owl:priorVersion	∈ IOXP	⊆ IX × IX
owl:propertyChainAxiom	∈ IP	⊆ IP × ISEQ
owl:propertyDisjointWith	∈ IP	⊆ IP × IP
owl:qualifiedCardinality	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × INNI
owl:sameAs	∈ IP	⊆ IR × IR
rdfs:seeAlso	∈ IOAP	⊆ IR × IR
owl:someValuesFrom	∈ IP	⊆ ICEXT(*I*(owl:Restriction)) × IC

owl:sourceIndividual	∈ IP	⊆ ICEXT(*I*(owl:NegativePropertyAssertion)) × IR
owl:targetIndividual	∈ IP	⊆ ICEXT(*I*(owl:NegativePropertyAssertion)) × IR
owl:targetValue	∈ IP	⊆ ICEXT(*I*(owl:NegativePropertyAssertion)) × LV
owl:topDataProperty	∈ IODP	= IR × LV
owl:topObjectProperty	∈ IP	= IR × IR
owl:unionOf	∈ IP	⊆ IC × ISEQ
owl:versionInfo	∈ IOAP	⊆ IR × IR
owl:versionIRI	∈ IOXP	⊆ IX × IX
owl:withRestrictions	∈ IP	⊆ IDC × ISEQ

5.4 Semantic Conditions for Boolean Connectives

Table 5.4 lists the semantic conditions for boolean connectives, including intersections, unions and complements of classes and datatypes. An intersection or a union of a collection of datatypes or a complement of a datatype is itself a datatype. While a complement of a class is created w.r.t. the whole universe, a datatype complement is created for a datatype w.r.t. the set of data values only.

Informative notes: Every first semantic condition of the three condition pairs in the table is an "iff" condition, since the corresponding OWL 2 language constructs are both class expressions and axioms. In contrast, the semantic condition on datatype complements is an "if-then" condition, since it only corresponds to a datarange expression. See the *notes on the form of semantic conditions* for further information. For the remaining semantic conditions that treat the cases of intersections and unions of datatypes it is sufficient to have "if-then" conditions, since stronger "iff" conditions would be redundant due to the more general "iff" conditions that already exist for classes. Note that the datatype related semantic conditions do not apply to empty sets, but one can still receive a datatype from an empty set by explicitly asserting the resulting class to be an instance of class rdfs:Datatype.

Table 5.4: Semantic Conditions for Boolean Connectives

if s sequence of c_1 , … , c_n ∈ IR then		
$\langle z , s \rangle \in$ IEXT(I(`owl:intersectionOf`))	**iff**	$z , c_1 , \ldots , c_n \in$ IC , ICEXT(z) = ICEXT(c_1) ∩ … ∩ ICEXT(c_n)
if		**then**
s sequence of $d_1 , \ldots , d_n \in$ IDC , $n \geq 1$, $\langle z , s \rangle \in$ IEXT(I(`owl:intersectionOf`))		$z \in$ IDC
if s sequence of c_1 , … , c_n ∈ IR then		
$\langle z , s \rangle \in$ IEXT(I(`owl:unionOf`))	**iff**	$z , c_1 , \ldots , c_n \in$ IC , ICEXT(z) = ICEXT(c_1) ∪ … ∪ ICEXT(c_n)
if		**then**
s sequence of $d_1 , \ldots , d_n \in$ IDC , $n \geq 1$, $\langle z , s \rangle \in$ IEXT(I(`owl:unionOf`))		$z \in$ IDC
$\langle z , c \rangle \in$ IEXT(I(`owl:complementOf`))	**iff**	$z , c \in$ IC , ICEXT(z) = IR \ ICEXT(c)
if		**then**
$\langle z , d \rangle \in$ IEXT(I(`owl:datatypeComplementOf`))		ICEXT(z) = LV \ ICEXT(d)

5.5 Semantic Conditions for Enumerations

Table 5.5 lists the semantic conditions for enumerations, i.e. classes that consist of an explicitly given finite set of instances. In particular, an enumeration entirely consisting of data values is a datatype.

Informative notes: The first semantic condition is an "iff" condition, since the corresponding OWL 2 language construct is both a class expression and an axiom. See the *notes on the form of semantic conditions* for further information. For the remaining semantic condition that treats the case of enumerations of data values it is sufficient to have an "if-then" condition, since a stronger "iff" condition would be redundant due to the more general "iff" condition that already exists for individuals.

Note that the data value related semantic condition does not apply to empty sets, but one can still receive a datatype from an empty set by explicitly asserting the resulting class to be an instance of class `rdfs:Datatype`.

Table 5.5: Semantic Conditions for Enumerations

if *s* sequence of a_1 , … , a_n ∈ IR then		
⟨ *z* , *s* ⟩ ∈ IEXT(*I*(`owl:oneOf`))	**iff**	*z* ∈ IC , ICEXT(*z*) = { a_1 , … , a_n }

if	**then**
s sequence of v_1 , … , v_n ∈ LV , *n* ≥ 1 , ⟨ *z* , *s* ⟩ ∈ IEXT(*I*(`owl:oneOf`))	*z* ∈ IDC

5.6 Semantic Conditions for Property Restrictions

Table 5.6 lists the semantic conditions for property restrictions.

Value restrictions require that some or all of the values of a certain property must be instances of a given class, or that the property has a specifically defined value. By placing a *self restriction* on some given property one only considers those individuals that are reflexively related to themselves via this property. *Cardinality restrictions* determine how often a certain property is allowed to be applied to a given individual. *Qualified cardinality restrictions* are more specific than cardinality restrictions in that they determine the quantity of a property application with respect to a particular class from which the property values are taken.

Implementations are *not* required to support the semantic conditions for `owl:onProperties`, but *may* support them in order to realize *n-ary dataranges* with arity ≥ 2 (see Section 7 of [*OWL 2 Specification*] for further information).

Informative notes: All the semantic conditions are "if-then" conditions, since the corresponding OWL 2 language constructs are class expressions. The "if-then" conditions generally only list those consequences on their right hand side that are specific for the respective condition, i.e. consequences that do not already follow by other means. See the *notes on the form of semantic conditions* for further information. Note that the semantic condition for *self restrictions* does not constrain the right hand side of a `owl:hasSelf` assertion to be the boolean value `"true"^^xsd:boolean`. See Section 5.3 for an explanation.

Table 5.6: Semantic Conditions for Property Restrictions

if	**then**
⟨ *z* , *c* ⟩ ∈ IEXT(*I*(`owl:someValuesFrom`)) , ⟨ *z* , *p* ⟩ ∈ IEXT(*I*(`owl:onProperty`))	ICEXT(*z*) = { *x* | ∃ *y* : ⟨ *x* , *y* ⟩ ∈ IEXT(*p*) and *y* ∈ ICEXT(*c*) }

s sequence of p_1 , … , $p_n \in$ IR , $n \geq 1$, $\langle z , c \rangle \in$ IEXT(I(owl:someValuesFrom)) , $\langle z , s \rangle \in$ IEXT(I(owl:onProperties))	p_1 , … , $p_n \in$ IP , ICEXT(z) = { x | $\exists$ y_1 , … , y_n : $\langle x , y_k \rangle \in$ IEXT(p_k) for each $1 \leq k \leq n$ and $\langle y_1 , … , y_n \rangle \in$ ICEXT(c) }
$\langle z , c \rangle \in$ IEXT(I(owl:allValuesFrom)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))	ICEXT(z) = { x | $\forall$ y : $\langle x , y \rangle \in$ IEXT(p) implies $y \in$ ICEXT(c) }
s sequence of p_1 , … , $p_n \in$ IR , $n \geq 1$, $\langle z , c \rangle \in$ IEXT(I(owl:allValuesFrom)) , $\langle z , s \rangle \in$ IEXT(I(owl:onProperties))	p_1 , … , $p_n \in$ IP , ICEXT(z) = { x | $\forall$ y_1 , … , y_n : $\langle x , y_k \rangle \in$ IEXT(p_k) for each $1 \leq k \leq n$ implies $\langle y_1 , … , y_n \rangle \in$ ICEXT(c) }
$\langle z , a \rangle \in$ IEXT(I(owl:hasValue)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))	ICEXT(z) = { x | $\langle x , a \rangle \in$ IEXT(p) }
$\langle z , v \rangle \in$ IEXT(I(owl:hasSelf)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))	ICEXT(z) = { x | $\langle x , x \rangle \in$ IEXT(p) }
$\langle z , n \rangle \in$ IEXT(I(owl:minCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))	ICEXT(z) = { x | #{ y | $\langle x , y \rangle \in$ IEXT(p) } $\geq n$ }
$\langle z , n \rangle \in$ IEXT(I(owl:maxCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))	ICEXT(z) = { x | #{ y | $\langle x , y \rangle \in$ IEXT(p) } $\leq n$ }
$\langle z , n \rangle \in$ IEXT(I(owl:cardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))	ICEXT(z) = { x | #{ y | $\langle x , y \rangle \in$ IEXT(p) } = n }
$\langle z , n \rangle \in$ IEXT(I(owl:minQualifiedCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty)) , $\langle z , c \rangle \in$ IEXT(I(owl:onClass))	ICEXT(z) = { x | #{ y | $\langle x , y \rangle \in$ IEXT(p) and $y \in$ ICEXT(c) } $\geq n$ }
$\langle z , n \rangle \in$ IEXT(I(owl:minQualifiedCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty)) , $\langle z , d \rangle \in$ IEXT(I(owl:onDataRange))	$p \in$ IODP , ICEXT(z) = { x | #{ $y \in$ LV | $\langle x , y \rangle \in$ IEXT(p) and $y \in$ ICEXT(d) } $\geq n$ }
$\langle z , n \rangle \in$ IEXT(I(owl:maxQualifiedCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty)) , $\langle z , c \rangle \in$ IEXT(I(owl:onClass))	ICEXT(z) = { x | #{ y | $\langle x , y \rangle \in$ IEXT(p) and $y \in$ ICEXT(c) } $\leq n$ }

$\langle z , n \rangle \in$ IEXT(I(owl:maxQualifiedCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty)) , $\langle z , d \rangle \in$ IEXT(I(owl:onDataRange))	$p \in$ IODP , ICEXT(z) = { x | #{ $y \in$ LV | $\langle x , y \rangle \in$ IEXT(p) and $y \in$ ICEXT(d) } $\leq n$ }
$\langle z , n \rangle \in$ IEXT(I(owl:qualifiedCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty)) , $\langle z , c \rangle \in$ IEXT(I(owl:onClass))	ICEXT(z) = { x | #{ y | $\langle x , y \rangle \in$ IEXT(p) and $y \in$ ICEXT(c) } = n }
$\langle z , n \rangle \in$ IEXT(I(owl:qualifiedCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty)) , $\langle z , d \rangle \in$ IEXT(I(owl:onDataRange))	$p \in$ IODP , ICEXT(z) = { x | #{ $y \in$ LV | $\langle x , y \rangle \in$ IEXT(p) and $y \in$ ICEXT(d) } = n }

5.7 Semantic Conditions for Datatype Restrictions

Table 5.7 lists the semantic conditions for datatype restrictions, which are used to define sub datatypes of existing datatypes by restricting the original datatype by means of a set of facet-value pairs. For information and an example on facets (see Section 3.4).

Certain special cases exist: If no facet-value pair is applied to a given datatype at all, then the resulting datatype will be equivalent to the original datatype. Further, if a facet-value pair is applied to a datatype without being a member of the datatype's facet space, then the ontology cannot be satisfied and will therefore be inconsistent. In particular, a datatype restriction with one or more specified facet-value pairs will result in an inconsistent ontology, if applied to a datatype with an empty facet space.

The set **IFS(*d*)** for a datatype d is defined by IFS(d) := { $\langle I(F) , v \rangle$ \| $\langle F , v \rangle \in$ FS(d) } , where F is the IRI of a facet, and v is a value of the facet. This set corresponds to the facet space FS(d), as defined in Section 4.1, but rather consists of pairs of the *denotation* of a facet and its value.

The mapping **IF2V(*d*)** for a datatype d is defined by IF2V(d)($\langle I(F) , v \rangle$) := F2V(d)($\langle F , v \rangle$) , where F is the IRI of a facet, and v is a value of the facet. This mapping corresponds to the facet-to-value mapping F2V(d), as defined in Section 4.1, resulting in the same subsets of the value space VS(d), but rather applies to pairs of the *denotation* of a facet and its value.

Informative notes: The semantic condition is an "if-then" condition, since the corresponding OWL 2 language construct is a datarange expression. The "if-then" condition only lists those consequences on its right hand side that are specific for the condition, i.e. consequences that do not already follow by other means. See the *notes on the form of semantic conditions* for further information.

Table 5.7: Semantic Conditions for Datatype Restrictions

if	then
s sequence of z_1 , … , $z_n \in$ IR , f_1 , … , $f_n \in$ IP , $\langle z , d \rangle \in$ IEXT(I(`owl:onDatatype`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:withRestrictions`)) , $\langle z_1 , v_1 \rangle \in$ IEXT(f_1) , … , $\langle z_n , v_n \rangle \in$ IEXT(f_n)	$z , d \in$ IDC , f_1 , … , $f_n \in$ IODP , v_1 , … , $v_n \in$ LV , $\langle f_1 , v_1 \rangle$, … , $\langle f_n , v_n \rangle \in$ IFS(d) , ICEXT(z) = ICEXT(d) ∩ IF2V(d)($\langle f_1 , v_1 \rangle$) ∩ … ∩ IF2V($d$)($\langle f_n , v_n \rangle$)

5.8 Semantic Conditions for the RDFS Vocabulary

Table 5.8 *extends* the RDFS semantic conditions for subclass axioms, subproperty axioms, domain axioms and range axioms. The semantic conditions provided here are "iff" conditions, while the original semantic conditions, as specified in Section 4.1 of [*RDF Semantics*], were weaker "if-then" conditions. Only the additional semantic conditions are given here and the other conditions of RDF and RDFS are retained.

Informative notes: All the semantic conditions are "iff" conditions, since the corresponding OWL 2 language constructs are axioms. See the *notes on the form of semantic conditions* for further information.

Table 5.8: Semantic Conditions for the RDFS Vocabulary

$\langle c_1 , c_2 \rangle \in$ IEXT(I(`rdfs:subClassOf`))	**iff**	$c_1 , c_2 \in$ IC , ICEXT(c_1) ⊆ ICEXT(c_2)
$\langle p_1 , p_2 \rangle \in$ IEXT(I(`rdfs:subPropertyOf`))		$p_1 , p_2 \in$ IP , IEXT(p_1) ⊆ IEXT(p_2)
$\langle p , c \rangle \in$ IEXT(I(`rdfs:domain`))		$p \in$ IP , $c \in$ IC , $\forall x , y : \langle x , y \rangle \in$ IEXT(p) implies $x \in$ ICEXT(c)
$\langle p , c \rangle \in$ IEXT(I(`rdfs:range`))		$p \in$ IP , $c \in$ IC , $\forall x , y : \langle x , y \rangle \in$ IEXT(p) implies $y \in$ ICEXT(c)

5.9 Semantic Conditions for Equivalence and Disjointness

Table 5.9 lists the semantic conditions for specifying that two individuals are equal or different from each other, and that either two classes or two properties are

equivalent or disjoint with each other, respectively. Also treated here are disjoint union axioms.

Informative notes: All the semantic conditions are "iff" conditions, since the corresponding OWL 2 language constructs are axioms. See the *notes on the form of semantic conditions* for further information. Also note that the IRI `owl:equivalentClass` is used to formulate *datatype definitions* (see Section 9.4 of [*OWL 2 Specification*] for information about datatype definitions).

Table 5.9: Semantic Conditions for Equivalence and Disjointness

$\langle a_1 , a_2 \rangle \in$ IEXT(I(`owl:sameAs`))	**iff**	$a_1 = a_2$
$\langle a_1 , a_2 \rangle \in$ IEXT(I(`owl:differentFrom`))		$a_1 \neq a_2$
$\langle c_1 , c_2 \rangle \in$ IEXT(I(`owl:equivalentClass`))		$c_1 , c_2 \in$ IC , ICEXT(c_1) = ICEXT(c_2)
$\langle c_1 , c_2 \rangle \in$ IEXT(I(`owl:disjointWith`))		$c_1 , c_2 \in$ IC , ICEXT(c_1) ∩ ICEXT(c_2) = $\emptyset$
$\langle p_1 , p_2 \rangle \in$ IEXT(I(`owl:equivalentProperty`))		$p_1 , p_2 \in$ IP , IEXT(p_1) = IEXT(p_2)
$\langle p_1 , p_2 \rangle \in$ IEXT(I(`owl:propertyDisjointWith`))		$p_1 , p_2 \in$ IP , IEXT(p_1) ∩ IEXT(p_2) = $\emptyset$
if s sequence of $c_1 , \ldots , c_n \in$ IR then		
$\langle c , s \rangle \in$ IEXT(I(`owl:disjointUnionOf`))	**iff**	$c , c_1 , \ldots , c_n \in$ IC , ICEXT(c) = ICEXT(c_1) ∪ … ∪ ICEXT(c_n) , ICEXT(c_j) ∩ ICEXT(c_k) = $\emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$

5.10 Semantic Conditions for N-ary Disjointness

Table 5.10 lists the semantic conditions for specifying n-ary diversity and disjointness axioms, i.e. that several given individuals are mutually different from each other, and that several given classes or properties are mutually disjoint with each other, respectively.

Note that there are two alternative ways to specify `owl:AllDifferent` axioms, by using either the property `owl:members` that is used for all other constructs, too,

or by applying the legacy property `owl:distinctMembers`. Both variants have an equivalent formal meaning.

Informative notes: The semantic conditions essentially represent "iff" conditions, since the corresponding OWL 2 language constructs are axioms. However, there are actually *two* semantic conditions for each language construct due to the multi-triple RDF encoding of these language constructs. The "if-then" conditions only list those consequences on their right hand side that are specific for the respective condition, i.e. consequences that do not already follow by other means. See the *notes on the form of semantic conditions* for further information.

Table 5.10: Semantic Conditions for N-ary Disjointness

if	**then**
s sequence of a_1 , … , $a_n \in$ IR , $z \in$ ICEXT(I(`owl:AllDifferent`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:members`))	$a_j \neq a_k$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
if	**then exists $z \in$ IR**
s sequence of a_1 , … , $a_n \in$ IR , $a_j \neq a_k$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$	$z \in$ ICEXT(I(`owl:AllDifferent`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:members`))
if	**then**
s sequence of a_1 , … , $a_n \in$ IR , $z \in$ ICEXT(I(`owl:AllDifferent`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:distinctMembers`))	$a_j \neq a_k$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
if	**then exists $z \in$ IR**
s sequence of a_1 , … , $a_n \in$ IR , $a_j \neq a_k$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$	$z \in$ ICEXT(I(`owl:AllDifferent`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:distinctMembers`))
if	**then**
s sequence of c_1 , … , $c_n \in$ IR , $z \in$ ICEXT(I(`owl:AllDisjointClasses`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:members`))	c_1 , … , $c_n \in$ IC , ICEXT(c_j) ∩ ICEXT(c_k) = $\emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$

if	then exists $z \in$ IR
s sequence of c_1 , … , $c_n \in$ IC , ICEXT(c_j) ∩ ICEXT(c_k) = ∅ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$	$z \in$ ICEXT(I(`owl:AllDisjointClasses`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:members`))

if	then
s sequence of p_1 , … , $p_n \in$ IR , $z \in$ ICEXT(I(`owl:AllDisjointProperties`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:members`))	p_1 , … , $p_n \in$ IP , IEXT(p_j) ∩ IEXT(p_k) = ∅ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$

if	then exists $z \in$ IR
s sequence of p_1 , … , $p_n \in$ IP , IEXT(p_j) ∩ IEXT(p_k) = ∅ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$	$z \in$ ICEXT(I(`owl:AllDisjointProperties`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:members`))

5.11 Semantic Conditions for Sub Property Chains

Table 5.11 lists the semantic conditions for sub property chains, which allow for specifying complex property subsumption axioms.

As an example, one can define a sub property chain axiom that specifies the chain consisting of the extensions of the properties `ex:hasFather` and `ex:hasBrother` to be a sub relation of the extension of the property `ex:hasUncle`.

Informative notes: The semantic condition is an "iff" condition, since the corresponding OWL 2 language construct is an axiom. See the *notes on the form of semantic conditions* for further information. The semantics has been specified in a way that allows a sub property chain axiom to be satisfiable without requiring the existence of a property that represents the property chain.

Table 5.11: Semantic Conditions for Sub Property Chains

if s sequence of p_1 , … , $p_n \in$ IR then		
$\langle p , s \rangle \in$ IEXT(I(`owl:propertyChainAxiom`))	**iff**	$p \in$ IP , p_1 , … , $p_n \in$ IP , $\forall y_0$, … , y_n : $\langle y_0 , y_1 \rangle \in$ IEXT(p_1) and … and $\langle y_{n-1} , y_n \rangle \in$

		IEXT(p_n) implies $\langle y_0 , y_n \rangle \in$ IEXT(p)

5.12 Semantic Conditions for Inverse Properties

Table 5.12 lists the semantic conditions for inverse property axioms. The inverse of a given property is the corresponding property with subject and object swapped for each property assertion built from it.

Informative notes: The semantic condition is an "iff" condition, since the corresponding OWL 2 language construct is an axiom. See the *notes on the form of semantic conditions* for further information.

Table 5.12: Semantic Conditions for Inverse Properties

$\langle p_1 , p_2 \rangle \in$ IEXT(I(`owl:inverseOf`))	**iff**	$p_1 , p_2 \in$ IP , IEXT(p_1) = { $\langle x , y \rangle$ | $\langle y , x \rangle \in$ IEXT(p_2) }

5.13 Semantic Conditions for Property Characteristics

Table 5.13 lists the semantic conditions for property characteristics.

If a property is *functional*, then at most one distinct value can be assigned to any given individual via this property. An *inverse functional* property can be regarded as a "key" property, i.e. no two different individuals can be assigned the same value via this property. A *reflexive* property relates every individual in the universe to itself, whereas an *irreflexive* property does not relate any individual with itself at all. If two individuals are related by a *symmetric* property, then this property also relates them reversely, while this is never the case for an *asymmetric* property. A *transitive* property that relates an individual *a* with an individual *b*, and the latter with an individual *c*, also relates *a* with *c*.

Informative notes: All the semantic conditions are "iff" conditions, since the corresponding OWL 2 language constructs are axioms. See the *notes on the form of semantic conditions* for further information.

Table 5.13: Semantic Conditions for Property Characteristics

$p \in$ ICEXT(I(`owl:FunctionalProperty`))	**iff**	$p \in$ IP , $\forall\, x , y_1 , y_2 : \langle x , y_1 \rangle \in$ IEXT(p) and $\langle x , y_2 \rangle \in$ IEXT(p) implies $y_1 = y_2$
$p \in$ ICEXT(I(`owl:InverseFunctionalProperty`))		$p \in$ IP , $\forall\, x_1 , x_2 , y : \langle x_1 , y \rangle \in$ IEXT(p) and $\langle x_2 , y \rangle \in$ IEXT(p) implies $x_1 = x_2$
$p \in$ ICEXT(I(`owl:ReflexiveProperty`))		$p \in$ IP , $\forall\, x : \langle x , x \rangle \in$ IEXT(p)
$p \in$ ICEXT(I(`owl:IrreflexiveProperty`))		$p \in$ IP , $\forall\, x : \langle x , x \rangle \notin$ IEXT(p)
$p \in$ ICEXT(I(`owl:SymmetricProperty`))		$p \in$ IP , $\forall\, x , y : \langle x , y \rangle \in$ IEXT(p) implies $\langle y , x \rangle \in$ IEXT(p)
$p \in$ ICEXT(I(`owl:AsymmetricProperty`))		$p \in$ IP , $\forall\, x , y : \langle x , y \rangle \in$ IEXT(p) implies $\langle y , x \rangle \notin$ IEXT(p)
$p \in$ ICEXT(I(`owl:TransitiveProperty`))		$p \in$ IP , $\forall\, x , y , z : \langle x , y \rangle \in$ IEXT(p) and $\langle y , z \rangle \in$ IEXT(p) implies $\langle x , z \rangle \in$ IEXT(p)

5.14 Semantic Conditions for Keys

Table 5.14 lists the semantic conditions for Keys.

Keys provide an alternative to inverse functional properties (see Section 5.13). They allow for defining a property as a key local to a given class: the specified property will have the features of a key only for individuals being instances of the class, and no assumption is made about individuals for which membership of the class cannot be entailed. Further, it is possible to define "compound keys", i.e. several properties can be combined into a single key applicable to composite values. Note that keys are not functional by default under the OWL 2 RDF-Based Semantics.

Informative notes: The semantic condition is an "iff" condition, since the corresponding OWL 2 language construct is an axiom. See the *notes on the form of semantic conditions* for further information.

Table 5.14: Semantic Conditions for Keys

if *s* sequence of p_1 , … , p_n ∈ IR then		
$\langle c , s \rangle \in$ IEXT(*I*(`owl:hasKey`))	**iff**	$c \in$ IC , p_1 , … , $p_n \in$ IP , $\forall x , y , z_1 , \ldots , z_n$: if $x \in$ ICEXT(c) and $y \in$ ICEXT(c) and $\langle x , z_k \rangle \in$ IEXT(p_k) and $\langle y , z_k \rangle \in$ IEXT(p_k) for each $1 \leq k \leq n$ then $x = y$

5.15 Semantic Conditions for Negative Property Assertions

Table 5.15 lists the semantic conditions for negative property assertions. They allow to state that two given individuals are *not* related by a given property.

The second form based on `owl:targetValue` is more specific than the first form based on `owl:targetIndividual` in that it is restricted to the case of negative *data* property assertions. Note that the second form will coerce the target individual of a negative property assertion into a data value, due to the range defined for the property `owl:targetValue` in Section 5.3.

Informative notes: The semantic conditions essentially represent "iff" conditions, since the corresponding OWL 2 language constructs are axioms. However, there are actually *two* semantic conditions for each language construct, due to the multi-triple RDF encoding of these language constructs. The "if-then" conditions only list those consequences on their right hand side that are specific for the respective condition, i.e. consequences that do not already follow by other means. See the *notes on the form of semantic conditions* for further information.

Table 5.15: Semantic Conditions for Negative Property Assertions

if	**then**
$\langle z , a_1 \rangle \in$ IEXT(*I*(`owl:sourceIndividual`)) , $\langle z , p \rangle \in$ IEXT(*I*(`owl:assertionProperty`)) , $\langle z , a_2 \rangle \in$ IEXT(*I*(`owl:targetIndividual`))	$\langle a_1 , a_2 \rangle \notin$ IEXT(p)
if	**then exists $z \in$ IR**

$a_1 \in$ IR , $p \in$ IP , $a_2 \in$ IR , $\langle a_1 , a_2 \rangle \notin$ IEXT(p)	$\langle z , a_1 \rangle \in$ IEXT(I(`owl:sourceIndividual`)) , $\langle z , p \rangle \in$ IEXT(I(`owl:assertionProperty`)) , $\langle z , a_2 \rangle \in$ IEXT(I(`owl:targetIndividual`))

if	**then**
$\langle z , a \rangle \in$ IEXT(I(`owl:sourceIndividual`)) , $\langle z , p \rangle \in$ IEXT(I(`owl:assertionProperty`)) , $\langle z , v \rangle \in$ IEXT(I(`owl:targetValue`))	$p \in$ IODP , $\langle a , v \rangle \notin$ IEXT(p)

if	**then exists $z \in$ IR**
$a \in$ IR , $p \in$ IODP , $v \in$ LV , $\langle a , v \rangle \notin$ IEXT(p)	$\langle z , a \rangle \in$ IEXT(I(`owl:sourceIndividual`)) , $\langle z , p \rangle \in$ IEXT(I(`owl:assertionProperty`)) , $\langle z , v \rangle \in$ IEXT(I(`owl:targetValue`))

6 Appendix: Axiomatic Triples (Informative)

The RDF Semantics specification [*RDF Semantics*] defines so called *"axiomatic triples"* as part of the semantics of RDF and RDFS. Unlike the RDF Semantics, the OWL 2 RDF-Based Semantics does not normatively specify any axiomatic triples. It might not be possible to give a set of RDF triples that captures all "axiomatic aspects" of the OWL 2 RDF-Based Semantics, just as one cannot expect to define the whole OWL 2 RDF-Based Semantics specification in terms of RDF entailment rules only. Furthermore, axiomatic triples for the OWL 2 RDF-Based Semantics could, in principle, contain arbitrarily complex class expressions, e.g. the union of several classes, and by this it becomes non-obvious which of several possible non-equivalent sets of axiomatic triples should be selected. However, the OWL 2 RDF-Based Semantics includes many semantic conditions that can in a sense be regarded as being "axiomatic", and thus can be considered a replacement for the missing axiomatic triples. After an overview on axiomatic triples for RDF and RDFS in Section 6.1, the Sections 6.2 and 6.3 will discuss how the "axiomatic" semantic conditions of the OWL 2 RDF-Based Semantics relate to axiomatic triples, resulting in an example set of axiomatic triples that is compatible with the OWL 2 RDF-Based Semantics.

6.1 Axiomatic Triples in RDF

In RDF and RDFS [*RDF Semantics*], axiomatic triples are used to provide basic meaning for all the vocabulary terms of the two languages. This formal meaning is independent of any given RDF graph, and it even holds for vocabulary terms, which do not occur in a graph that is interpreted by an RDF or RDFS interpretation. As a consequence, all the axiomatic triples of RDF and RDFS are entailed by the *empty* graph, when being interpreted under the semantics of RDF or RDFS, respectively.

Examples of RDF and RDFS axiomatic triples are:

(1) `rdf:type rdf:type rdf:Property .`
(2) `rdf:type rdfs:domain rdfs:Resource .`
(3) `rdf:type rdfs:range rdfs:Class .`
(4) `rdfs:Datatype rdfs:subClassOf rdfs:Class .`
(5) `rdfs:isDefinedBy rdfs:subPropertyOf rdfs:seeAlso .`

As shown by these examples, axiomatic triples are typically used by the RDF Semantics specification to determine the part of the universe the denotation of a vocabulary term belongs to (1). In the case of a property, the domain (2) and range (3) is specified as well. Also, in some cases, hierarchical relationships between classes (4) or properties (5) of the vocabulary are determined.

Under the OWL 2 RDF-Based Semantics, all the axiomatic triples of RDF and RDFS could, in principle, be replaced by "axiomatic" semantic conditions that have neither premises nor bound variables. By specifically applying the *RDFS semantic conditions* given in Section 5.8, the example axiomatic triples (1) – (5) can be equivalently restated as:

I(`rdf:type`) ∈ ICEXT(I(`rdf:Property`)) ,
IEXT(I(`rdf:type`)) ⊆ ICEXT(I(`rdfs:Resource`)) × ICEXT(I(`rdfs:Class`)) ,
ICEXT(I(`rdfs:Datatype`)) ⊆ ICEXT(I(`rdfs:Class`)) ,
IEXT(I(`rdfs:isDefinedBy`)) ⊆ IEXT(I(`rdfs:seeAlso`)) .

All the axiomatic triples of RDF and RDFS can be considered *"simple"* in the sense that they have in their object position only single terms from the RDF and RDFS vocabularies, and no complex class or property expressions appear there.

6.2 Axiomatic Triples for the Vocabulary Classes

The semantic conditions for *vocabulary classes* in Table 5.2 of Section 5.2 can be considered as corresponding to a set of axiomatic triples for the classes in the vocabulary of the OWL 2 RDF-Based Semantics.

First, for each IRI *E* occurring in the first column of Table 5.2, if the *second* column contains an entry of the form "I(*E*) ∈ *S*" for some set *S*, then this entry corresponds to some RDF triple of the form "*E* `rdf:type` *C*", where *C* is the IRI of some class

with ICEXT($I(C)$) = S. In the table, S will always be either the part IC of all classes, or some sub part of IC. Hence, in a corresponding RDF triple the IRI C will typically be one of "`rdfs:Class`", "`owl:Class`" (S=IC in both cases) or "`rdfs:Datatype`" (S=IDC).

For example, the semantic condition for the IRI "`owl:FunctionalProperty`", given by

I(`owl:FunctionalProperty`) ∈ IC ,

would have the corresponding axiomatic triple

```
owl:FunctionalProperty rdf:type rdfs:Class .
```

Further, for each IRI E in the first column of the table, if the *third* column contains an entry of the form "ICEXT($I(E)$) ⊆ S" (or "ICEXT($I(E)$) = S") for some set S, then this entry corresponds to some RDF triple of the form "E `rdfs:subClassOf` C" (or "E `owl:equivalentClass` C"), where C is the IRI of some class with ICEXT($I(C)$) = S. In every case, S will be either one of the parts of the universe of I or the empty set.

For example, the semantic condition

ICEXT(I(`owl:FunctionalProperty`)) ⊆ IP

would have the corresponding axiomatic triple

```
owl:FunctionalProperty rdfs:subClassOf rdf:Property .
```

In addition, the semantic conditions for the *parts of the universe* in Table 5.1 of Section 5.1 have to be taken into account. In particular, if an entry in the *second* column of Table 5.1 is of the form "$S_1 \subseteq S_2$" for some sets S_1 and S_2, then this corresponds to some RDF triple of the form "C_1 `owl:subClassOf` C_2", where C_1 and C_2 are the IRIs of some classes with ICEXT($I(C_1)$) = S_1 and ICEXT($I(C_2)$) = S_2, respectively, according to Section 5.2.

As stated in Section 6.1 for the RDF axiomatic triples, all the axiomatic triples for classes can be considered *"simple"* in the sense that they will have in their object position only single terms from the RDF, RDFS and OWL 2 RDF-Based vocabularies (Section 3.2).

Note that some of the axiomatic triples obtained in this way already follow from the semantics of RDF and RDFS, as defined in [*RDF Semantics*].

6.3 Axiomatic Triples for the Vocabulary Properties

The semantic conditions for *vocabulary properties* in Table 5.3 of Section 5.3 can be considered as corresponding to a set of axiomatic triples for the properties in the vocabulary of the OWL 2 RDF-Based Semantics.

First, for each IRI *E* occurring in the first column of Table 5.3, if the *second* column contains an entry of the form "$I(E) \in S$" for some set *S*, then this entry corresponds to some RDF triple of the form "*E* `rdf:type` *C*", where *C* is the IRI of some class with ICEXT($I(C)$) = *S*. In the table, *S* will always be either the part IP of all properties, or some sub part of IP. Hence, in a corresponding RDF triple the IRI *C* will typically be one of "`rdf:Property`", "`owl:ObjectProperty`", (*S*=IP in both cases), "`owl:DatatypeProperty`" (*S*=IODP), "`owl:OntologyProperty`" (*S*=IOXP) or "`owl:AnnotationProperty`" (*S*=IOAP).

For example, the semantic condition for the IRI "`owl:disjointWith`", given by

I(`owl:disjointWith`) $\in$ IP ,

would have the corresponding axiomatic triple

```
owl:disjointWith rdf:type rdf:Property .
```

Further, for each IRI *E* in the first column of the table, if the *third* column contains an entry of the form "IEXT($I(E)$) $\subseteq S_1 \times S_2$" for some sets S_1 and S_2, then this entry corresponds to some RDF triples of the forms "*E* `rdfs:domain` C_1" and "*E* `rdfs:range` C_2", where C_1 and C_2 are the IRIs of some classes with ICEXT($I(C_1)$) = S_1 and ICEXT($I(C_2)$) = S_2, respectively. Note that the sets S_1 and S_2 do *not always* correspond to any of the parts of the universe of *I*.

For example, the semantic condition

IEXT(I(`owl:disjointWith`)) $\subseteq$ IC $\times$ IC

would have the corresponding axiomatic triples

```
owl:disjointWith rdfs:domain owl:Class .
owl:disjointWith rdfs:range owl:Class .
```

Exceptions are the semantic conditions "IEXT(I(`owl:topObjectProperty`)) = IR $\times$ IR" and "IEXT(I(`owl:topDataProperty`)) = IR $\times$ LV", since the *exactly* specified property extensions of these properties cannot be expressed solely by domain and range axiomatic triples. For example, the domain and range axiomatic triples for `owl:sameAs` are equal to those for `owl:topObjectProperty`, but the property extension of `owl:sameAs` is different from that of `owl:topObjectProperty`.

As stated in Section 6.1 for the RDF axiomatic triples, all the axiomatic triples for properties can be considered *"simple"* in the sense that they will have in their object position only single terms from the RDF, RDFS and OWL 2 RDF-Based vocabularies (Section 3.2).

7 Appendix: Relationship to the Direct Semantics (Informative)

This section compares the OWL 2 RDF-Based Semantics with the *OWL 2 Direct Semantics* [*OWL 2 Direct Semantics*]. While the OWL 2 RDF-Based Semantics is based on the RDF Semantics specification [*RDF Semantics*], the OWL 2 Direct Semantics is a *description logic* style semantics. Several fundamental differences exist between the two semantics, but there is also a strong relationship basically stating that the OWL 2 RDF-Based Semantics is able to reflect all logical conclusions of the OWL 2 Direct Semantics. This means that the OWL 2 Direct Semantics can in a sense be regarded as a sub semantics of the OWL 2 RDF-Based Semantics.

Technically, the comparison will be performed by comparing the sets of *entailments* that hold for each of the two semantics, respectively. The definition of an ***OWL 2 RDF-Based entailment*** was given in Section 4.3 of this document, while the definition of an ***OWL 2 Direct entailment*** is provided in Section 2.5 of [*OWL 2 Direct Semantics*]. In both cases, entailments are defined for pairs of ontologies, and such an ordered pair of two ontologies will be called an ***entailment query*** in this section.

Comparing the two semantics by means of entailments will only be meaningful if the entailment queries allow for applying both the OWL 2 RDF-Based Semantics and the OWL 2 Direct Semantics to them. In order to ensure this, the comparison will be restricted to entailment queries, for which the left-hand side and right-hand side ontologies are both ***OWL 2 DL ontologies in RDF graph form***. These are RDF graphs that can be transformed by applying the *reverse OWL 2 RDF mapping* [*OWL 2 RDF Mapping*] into corresponding ***OWL 2 DL ontologies in Functional Syntax form*** according to the *Functional style syntax* defined in [*OWL 2 Specification*], and which must further meet all the *restrictions on OWL 2 DL ontologies* that are specified in Section 3 of [*OWL 2 Specification*]. In fact, these restrictions must be *mutually met* by both ontologies that occur in an entailment query, i.e. all these restrictions need to be satisfied as if the two ontologies would be part of a single ontology. Any entailment query that adheres to the conditions defined here will be called an ***OWL 2 DL entailment query***.

Ideally, the relationship between the OWL 2 RDF-Based Semantics and the OWL 2 Direct Semantics would be of the form that every OWL 2 DL entailment query that is an OWL 2 Direct entailment is also an OWL 2 RDF-Based entailment. However, this desirable relationship cannot hold in general due to a variety of differences that exist between the OWL 2 RDF-Based Semantics and the OWL 2 Direct Semantics, as demonstrated in Section 7.1.

Fortunately, the problems resulting from these semantic differences can be overcome in a way that for every OWL 2 DL entailment query there is another one for which the desired entailment relationship indeed holds, and the new entailment query is semantically equivalent to the original entailment query under the OWL 2 Direct Semantics. This is the gist of the *OWL 2 Correspondence Theorem*, which will be presented in Section 7.2. The *proof* of this theorem, given in Section 7.3, will further demonstrate that such a substitute OWL 2 DL entailment query can always be algorithmically constructed by means of simple syntactic transformations.

7.1 Example on Semantic Differences

This section will show that differences exist between the OWL 2 RDF-Based Semantics and the OWL 2 Direct Semantics, and it will be demonstrated how these semantic differences complicate a comparison of the two semantics in terms of entailments. An example OWL 2 DL entailment query will be given, which will happen to be an OWL 2 Direct entailment without being an OWL 2 RDF-Based entailment. The section will explain the different reasons and will provide a resolution of each of them. It will turn out that the example entailment query can be syntactically transformed into another OWL 2 DL entailment query that is both an OWL 2 Direct entailment and an OWL 2 RDF-Based entailment, while being semantically unchanged compared to the original entailment query under the OWL 2 Direct Semantics. This example will motivate the *OWL 2 Correspondence Theorem* in Section 7.2 and its proof in Section 7.3.

The example entailment query consists of the following pair $\langle G_1^*, G_2^* \rangle$ of RDF graphs:

G_1^*:

```
(1) ex:o1 rdf:type owl:Ontology .
(2) ex:c1 rdf:type owl:Class .
(3) ex:c2 rdf:type owl:Class .
(4) ex:c1 rdfs:subClassOf ex:c2 .
```

G_2^*:

```
(1) ex:o2 rdf:type owl:Ontology .
(2) ex:c1 rdf:type owl:Class .
(3) ex:c2 rdf:type owl:Class .
(4) ex:c3 rdf:type owl:Class .
(5) ex:c1 rdfs:subClassOf _:x .
(6) _:x rdf:type owl:Class .
(7) _:x owl:unionOf ( ex:c2 ex:c3 ) .
(8) ex:c3 rdfs:label "c3" .
```

Both G_1^* and G_2^* are OWL 2 DL ontologies in RDF graph form and can therefore be mapped by the reverse RDF mapping [*OWL 2 RDF Mapping*] to the following two OWL 2 DL ontologies in Functional Syntax form F(G_1^*) and F(G_2^*):

F(G_1^*) :

```
(1) Ontology( ex:o1
(2)   Declaration( Class( ex:c1 ) )
(3)   Declaration( Class( ex:c2 ) )
(4)   SubClassOf( ex:c1 ex:c2 )
(5) )
```

F(G_2^*) :

```
(1) Ontology( ex:o2
(2)   Declaration( Class( ex:c1 ) )
(3)   Declaration( Class( ex:c2 ) )
(4)   Declaration( Class( ex:c3 ) )
(5)   SubClassOf( ex:c1 ObjectUnionOf( ex:c2 ex:c3 ) )
(6)   AnnotationAssertion( rdfs:label ex:c3 "c3" )
(7) )
```

It follows that F(G_1^*) OWL 2 Direct entails F(G_2^*). To show this, only the axioms (4) of F(G_1^*) and (5) of F(G_2^*) have to be considered. None of the other statements in the two ontologies are relevant for this OWL 2 Direct entailment to hold, since they do not have a formal meaning under the OWL 2 Direct Semantics. However, it turns out that the RDF graph G_1^* does *not* OWL 2 RDF-Based entail G_2^*, for reasons discussed in detail now.

Reason 1: An annotation in F(G_2^*). The ontology F(G_2^*) contains an annotation (6). The OWL 2 Direct Semantics does not give a formal meaning to annotations. In contrast, under the OWL 2 RDF-Based Semantics *every* RDF triple occurring in an RDF graph has a formal meaning, including the corresponding annotation triple (8) in G_2^*. Since this annotation triple only occurs in G_2^* but not in G_1^*, there will exist OWL 2 RDF-Based interpretations that satisfy G_1^* without satisfying triple (8) of G_2^*. Hence, G_1^* does *not* OWL 2 RDF-Based entail G_2^*.

Resolution of Reason 1. The annotation triple (8) in G_2^* will be removed, which will avoid requiring OWL 2 RDF-Based interpretations to interpret this triple. The changed RDF graphs will still be OWL 2 DL ontologies in RDF graph form, since annotations are strictly optional in OWL 2 DL ontologies. Also, this operation will not change the formal meaning of the ontologies under the OWL 2 Direct Semantics, since annotations do not have a formal meaning under this semantics.

Reason 2: An entity declaration exclusively in F(G_2^*). The ontology F(G_2^*) contains an entity declaration for the class IRI `ex:c3` (4), for which there is no corresponding entity declaration in F(G_1^*). The OWL 2 Direct Semantics does not give a formal meaning to entity declarations, while the OWL 2 RDF-Based

Semantics gives a formal meaning to the corresponding declaration triple (4) in G_2^*. The consequences are analog to those described for reason 1.

Resolution of Reason 2. The declaration triple (4) in G_2^* will be copied to G_1^*. An OWL 2 RDF-Based interpretation that satisfies the modified graph G_1^* will then also satisfy the declaration triple. The changed RDF graphs will still be OWL 2 DL ontologies in RDF graph form, since adding the entity declaration does not hurt any of the restrictions on OWL 2 DL ontologies. Also, this operation will not change the formal meaning of the ontologies under the OWL 2 Direct Semantics, since entity declarations do not have a formal meaning under this semantics.

Reason 3: Different ontology IRIs in F(G_1^*) and F(G_2^*). The ontology IRIs for the two ontologies, given by (1) in F(G_1^*) and by (1) in F(G_2^*), differ from each other. The OWL 2 Direct Semantics does not give a formal meaning to ontology headers, while the OWL 2 RDF-Based Semantics gives a formal meaning to the corresponding header triples (1) in G_1^* and (1) in G_2^*. Since these header triples differ from each other, the consequences are analog to those described for reason 1.

Resolution of Reason 3. The IRI in the subject position of the header triple (1) in G_2^* is changed into a blank node. Due to the existential semantics of blank nodes under the OWL 2 RDF-Based Semantics this new triple will then be entailed by triple (1) in G_1^*. The changed RDF graphs will still be OWL 2 DL ontologies in RDF graph form, since an ontology IRI is optional for an OWL 2 DL ontology. (Note, however, that it would have been an error to simply remove triple (1) from G_2^*, since an OWL 2 DL ontology is required to contain an ontology header.) Also, this operation will not change the formal meaning of the ontologies under the OWL 2 Direct Semantics, since ontology headers do not have a formal meaning under this semantics.

Reason 4: A class expression in F(G_2^*). Axiom (5) of F(G_2^*) contains a class expression that represents the union of the two classes denoted by `ex:c2` and `ex:c3`. Within G_2^*, this class expression is represented by the triples (6) and (7), both having the blank node "`_:x`" in their respective subject position. The way the OWL 2 RDF-Based Semantics interprets these two triples differs from the way the OWL 2 Direct Semantics treats the class expression in axiom (5) of F(G_2^*).

The OWL 2 Direct Semantics treats classes as *sets*, i.e. subsets of the universe. Thus, the IRIs `ex:c2` and `ex:c3` in F(G_2^*) denote two sets, and the class expression in axiom (5) of F(G_2^*) therefore represents the set that consists of the union of these two sets.

The OWL 2 RDF-Based Semantics, on the other hand, treats classes as *individuals*, i.e. members of the universe. While every class under the OWL 2 RDF-Based Semantics represents a certain subset of the universe, namely its class extension, this set is actually distinguished from the class itself. For two given classes it is ensured under the OWL 2 RDF-Based Semantics, just as for the OWL 2 Direct Semantics, that the union of their class extensions will always exist as a

subset of the universe. However, there is no guarantee that there will also exist an individual in the universe that has this set union as its class extension.

Under the OWL 2 RDF-Based Semantics, triple (7) of G_2^* claims that a class exists being the union of two other classes. But since the existence of such a union class is not ensured by G_1^*, there will be OWL 2 RDF-Based interpretations that satisfy G_1^* without satisfying triple (7) of G_2^*. Hence, G_1^* does *not* OWL 2 RDF-Based entail G_2^*.

Resolution of Reason 4. The triples (6) and (7) of G_2^* are copied to G_1^* together with the new triple "`_:x owl:equivalentClass _:x`". If an OWL 2 RDF-Based interpretation satisfies the modified graph G_1^*, then the triples (6) and (7) of G_2^* will also be satisfied. The changed RDF graphs will still be OWL 2 DL ontologies in RDF graph form, since the whole set of added triples encodes a proper OWL 2 DL axiom. Further, for the IRI `ex:c3`, which occurs in the union class expression but not in G_1^*, an entity declaration is added to G_1^* by the resolution of reason 2. Also, this operation will not change the formal meaning of the ontologies under the OWL 2 Direct Semantics, since the added equivalence axiom is a tautology under this semantics.

Note that it would have been an error to simply copy the triples (6) and (7) of G_2^* to G_1^*, without also adding the new triple "`_:x owl:equivalentClass _:x`". This would have produced a class expression that has no connection to any axiom in the ontology. An OWL 2 DL ontology is basically a set of axioms and does not allow for the occurrence of "dangling" class expressions. This is the reason for actually "embedding" the class expression in an axiom. It would have also been wrong to use an *arbitrary* axiom for such an embedding, since it has to be ensured that the formal meaning of the original ontology does not change under the OWL 2 Direct Semantics. However, any *tautological* axiom that contains the original class expression would have been sufficient for this purpose as well.

Complete Resolution: The transformed entailment query.

Combining the resolutions of all the above reasons leads to the following new pair of RDF graphs $\langle G_1 , G_2 \rangle$:

G_1 :

```
(1) ex:o1 rdf:type owl:Ontology .
(2) ex:c1 rdf:type owl:Class .
(3) ex:c2 rdf:type owl:Class .
(4) ex:c3 rdf:type owl:Class .
(5) ex:c1 rdfs:subClassOf ex:c2 .
(6) _:x owl:equivalentClass _:x .
(7) _:x rdf:type owl:Class .
(8) _:x owl:unionOf ( ex:c2 ex:c3 ) .
```

G_2 :

```
(1) _:o rdf:type owl:Ontology .
(2) ex:c1 rdf:type owl:Class .
(3) ex:c2 rdf:type owl:Class .
(4) ex:c3 rdf:type owl:Class .
(5) ex:c1 rdfs:subClassOf _:x .
(6) _:x rdf:type owl:Class .
(7) _:x owl:unionOf ( ex:c2 ex:c3 ) .
```

The following list reiterates the changes compared to the original RDF graphs G_1^* and G_2^*:

- ***Resolution of Reason 1 (annotation):*** Triple (8) in G_2^* has been removed, i.e. there is no corresponding annotation triple in G_2.
- ***Resolution of Reason 2 (entity declaration):*** Triple (4) in G_2^* has been copied to G_1^*, becoming triple (4) in G_1.
- ***Resolution of Reason 3 (ontology IRIs):*** The IRI in the subject position of triple (1) in G_2^* has been changed into a blank node, becoming triple (1) in G_2.
- ***Resolution of Reason 4 (class expression):*** Triples (6) and (7) in G_2^* have been copied to G_1^* together with the new triple "`_:x owl:equivalentClass _:x`", becoming triples (6), (7) and (8) in G_1.

G_1 and G_2 are again OWL 2 DL ontologies in RDF graph form and can be mapped by the reverse RDF mapping to the following OWL 2 DL ontologies in Functional Syntax form F(G_1) and F(G_2):

F(G_1) :

```
(1) Ontology( ex:o1
(2)   Declaration( Class( ex:c1 ) )
(3)   Declaration( Class( ex:c2 ) )
(4)   Declaration( Class( ex:c3 ) )
(5)   SubClassOf( ex:c1 ex:c2 )
(6)   EquivalentClasses( ObjectUnionOf( ex:c2 ex:c3 )
ObjectUnionOf( ex:c2 ex:c3 ) )
(7) )
```

F(G_2) :

```
(1) Ontology(
(2)   Declaration( Class( ex:c1 ) )
(3)   Declaration( Class( ex:c2 ) )
(4)   Declaration( Class( ex:c3 ) )
(5)   SubClassOf( ex:c1 ObjectUnionOf( ex:c2 ex:c3 ) )
(6) )
```

As said earlier, all the applied changes preserve the formal meaning of the original OWL 2 DL ontologies under the OWL 2 Direct Semantics. Hence, it is still the case that F(G_1) OWL 2 Direct entails F(G_2). However, due to the syntactic transformation the situation has changed for the OWL 2 RDF-Based Semantics. It is now possible to show, by following the lines of argumentation for the resolutions of the different reasons given above, that G_1 OWL 2 RDF-Based entails G_2 as well.

7.2 Correspondence Theorem

This section presents the *OWL 2 Correspondence Theorem*, which compares the semantic expressivity of the OWL 2 RDF-Based Semantics with that of the OWL 2 Direct Semantics. The theorem basically states that the OWL 2 RDF-Based Semantics is able to reflect all the semantic conclusions of the OWL 2 Direct Semantics, where the notion of a "semantic conclusion" is technically expressed in terms of an *entailment*.

However, as discussed in Section 7.1, there exist semantic differences between the OWL 2 RDF-Based Semantics and the OWL 2 Direct Semantics, which do not allow for stating that *any* OWL 2 DL entailment query that is an OWL 2 Direct entailment will always also be an OWL 2 RDF-Based entailment. Nevertheless, it can still be ensured that any given OWL 2 DL entailment query can be *substituted* by another OWL 2 DL entailment query in a way that for the substitute entailment query the desired relationship will really hold, while preserving the formal meaning compared to the original entailment query under the OWL 2 Direct Semantics.

In fact, the theorem only makes the seemingly weak assertion that such a substitute entailment query will always *exist*. But the actual *proof of the theorem* in Section 7.3 will be more concrete in that it will substitute each given OWL 2 DL entailment query with a variant that can be algorithmically constructed by applying a set of simple syntactic transformations to the original entailment query. One can get an idea of how this works from Section 7.1.

Technical note on corresponding datatype maps:

A distinction exists between the format of an *OWL 2 RDF-Based datatype map*, as defined by Definition 4.1, and the format of an *OWL 2 Direct datatype map*, as defined in Section 2.1 of [*OWL 2 Direct Semantics*]. It is, however, possible to translate between an OWL 2 RDF-Based datatype map D and the corresponding OWL 2 Direct datatype map F(D) in the following way:

Let D be an OWL 2 RDF-Based datatype map according to Definition 4.1. The *corresponding OWL 2 Direct datatype map* F(D) := (N_{DT} , N_{LS} , N_{FS} , $\cdot^{DT}$, $\cdot^{LS}$, $\cdot^{FS}$) [*OWL 2 Direct Semantics*] is given by

- *Datatype Names:* N_{DT} is defined as the set of all IRIs u, for which there is a datatype d, such that $\langle u , d \rangle \in D$.
- *Lexical Space:* For each datatype name $u \in N_{DT}$, set $N_{LS}(u)$:= LS(d), where $\langle u , d \rangle \in D$.

- *Facet Space:* For each datatype name $u \in N_{DT}$, set $N_{FS(u)}$:= FS(d), where $\langle u , d \rangle \in D$.
- *Value Space:* For each datatype name $u \in N_{DT}$, set $(u)^{DT}$:= VS(d), where $\langle u , d \rangle \in D$.
- *Lexical-to-Value Mapping:* For each datatype name $u \in N_{DT}$ and each lexical form $a \in N_{LS(u)}$, set $\langle a , u \rangle^{LS}$:= L2V(d)(a), where $\langle u , d \rangle \in D$.
- *Facet-to-Value Mapping:* For each datatype name $u \in N_{DT}$ and each pair $\langle F , v \rangle \in N_{FS(u)}$, set $\langle F , v \rangle^{FS}$:= F2V(d)($\langle F , v \rangle$), where $\langle u , d \rangle \in D$.

Theorem 7.1 (OWL 2 Correspondence Theorem):

Let D be an OWL 2 RDF-Based datatype map according to Definition 4.1, with F(D) being the OWL 2 Direct datatype map according to Section 2.1 of [*OWL 2 Direct Semantics*] that corresponds to D according to the *technical note on corresponding datatype maps*. Let G_1^* and G_2^* be RDF graphs that are OWL 2 DL ontologies in RDF graph form, with F(G_1^*) and F(G_2^*) being the OWL 2 DL ontologies in Functional Syntax form [*OWL 2 Specification*] that result from applying the reverse OWL 2 RDF mapping [*OWL 2 RDF Mapping*] to G_1^* and G_2^*, respectively. Let F(G_1^*) and F(G_2^*) mutually meet the restrictions on OWL 2 DL ontologies as specified in Section 3 of [*OWL 2 Specification*].

Then, there exist RDF graphs G_1 and G_2 that are OWL 2 DL ontologies in RDF graph form, such that all the following conditions hold, with F(G_1) and F(G_2) being the OWL 2 DL ontologies in Functional Syntax form that result from applying the reverse OWL 2 RDF mapping to G_1 and G_2, respectively:

- (1) F(G_1) and F(G_2) mutually meet the restrictions on OWL 2 DL ontologies;
- (2) F(G_1) OWL 2 Direct entails F(G_1^*) with respect to $F(D)$, if and only if F(G_1^*) OWL 2 Direct entails F(G_1) with respect to $F(D)$;
- (3) F(G_2) OWL 2 Direct entails F(G_2^*) with respect to $F(D)$, if and only if F(G_2^*) OWL 2 Direct entails F(G_2) with respect to $F(D)$;
- (4) if F(G_1) OWL 2 Direct entails F(G_2) with respect to $F(D)$, then G_1 OWL 2 RDF-Based entails G_2 with respect to D.

7.3 Proof for the Correspondence Theorem

This is a sketch of a proof for *Theorem 7.1 (OWL 2 Correspondence Theorem)*, stated in Section 7.2. The proof sketch provides the basic line of argumentation for showing the theorem. However, for complexity reasons, some technical aspects of the theorem are only coarsely treated, and the proof sketch also refrains from taking the full amount of language constructs of OWL 2 into account. A complete proof can make use of the observation that the definitions of the OWL 2 Direct Semantics and the OWL 2 RDF-Based Semantics are actually closely aligned for all the different language constructs of OWL 2.

The proof sketch will make use of an approach that will be called *"balancing"* throughout this appendix, and which will now be introduced. A concrete example for how this approach can be applied is given in Section 7.1.

Definition (Balanced): A pair of RDF graphs $\langle G_1 , G_2 \rangle$ is called *balanced*, if and only if G_1 and G_2 are OWL 2 DL ontologies in RDF graph form, such that all the following additional conditions hold, with F(G_1) and F(G_2) being the OWL 2 DL ontologies in Functional Syntax form [OWL 2 Specification] that result from applying the reverse OWL 2 RDF mapping [OWL 2 RDF Mapping] to G_1 and G_2, respectively:

- (1) F(G_1) and F(G_2) mutually meet the restrictions on OWL 2 DL ontologies as specified in Section 3 of [OWL 2 Specification];
- (2) for every IRI or blank node *x* occurring in G_1 or G_2, there is a declaration triple (Table 7 in [OWL 2 RDF Mapping]) in the graph for every entity type (Section 5.8 of [OWL 2 Specification]) of *x* (there are no missing entity declarations);
- G_2 does not contain
  - (3) RDF encodings of annotations (Sections 3.2.2 and 3.2.3, and Table 17 in [OWL 2 RDF Mapping]);
  - (4) deprecation triples (Table 16 in [OWL 2 RDF Mapping]);
- (5) G_2 contains exactly one ontology header (Table 4 in [OWL 2 RDF Mapping]) consisting of a single RDF triple of the form "*b* `rdf:type owl:Ontology`", where *b* is a blank node;
- any subgraph *g* of G_2 that is the RDF encoding of one of the following OWL 2 constructs is also a subgraph of G_1:
  - (6) entity declaration (Table 7 in [OWL 2 RDF Mapping]);
  - (7) property expression (Table 11 in [OWL 2 RDF Mapping]);
  - (8) class expression (Tables 13 and 15 in [OWL 2 RDF Mapping]);
  - (9) data range expression (Tables 12 and 14 in [OWL 2 RDF Mapping]).

Balancing Lemma: An algorithm exists that terminates on every input and that has the following input/output behavior:

Let the *input* of the algorithm be a pair of RDF graphs $\langle G_1^* , G_2^* \rangle$, where G_1^* and G_2^* are OWL 2 DL ontologies in RDF graph form, with F(G_1^*) and F(G_2^*) being the OWL 2 DL ontologies in Functional Syntax form [OWL 2 Specification] that result from applying the reverse OWL 2 RDF mapping [OWL 2 RDF Mapping] to G_1^* and G_2^*, respectively. Let F(G_1^*) and F(G_2^*) mutually meet the restrictions on OWL 2 DL ontologies as specified in Section 3 of [OWL 2 Specification].

Then the *output* of the algorithm will be a pair of RDF graphs $\langle G_1 , G_2 \rangle$, where G_1 and G_2 are OWL 2 DL ontologies in RDF graph form, such that for any OWL 2 RDF-Based datatype map *D* according to Definition 4.1 all the following conditions hold, with F(G_1) and F(G_2) being the OWL 2 DL ontologies in Functional Syntax form that result from applying the reverse OWL 2 RDF mapping to G_1 and G_2, respectively, and with F(*D*) being the OWL 2 Direct datatype map according to Section 2.1 of [OWL 2 Direct Semantics] that corresponds to *D* according to the *technical note on corresponding datatype maps* in Section 7.2:

- (1) the pair $\langle G_1 , G_2 \rangle$ is *balanced*;

- (2) F(G_1) OWL 2 Direct entails F(G_1^*) with respect to *F(D)*, and F(G_1^*) OWL 2 Direct entails F(G_1) with respect to *F(D)*;
- (3) F(G_2) OWL 2 Direct entails F(G_2^*) with respect to *F(D)*, and F(G_2^*) OWL 2 Direct entails F(G_2) with respect to *F(D)*.

Proof of the Balancing Lemma:

Let G_1^* and G_2^* be OWL 2 DL ontologies in RDF graph form, with F(G_1^*) and F(G_2^*) being the corresponding OWL 2 DL ontologies in Functional Syntax form that result from applying the reverse OWL 2 RDF mapping to G_1^* and G_2^*, respectively, such that F(G_1^*) and F(G_2^*) mutually meet the restrictions on OWL 2 DL ontologies. The resulting RDF graphs G_1 and G_2 are constructed as follows.

The initial versions of G_1 and G_2 are copies of G_1^* and G_2^*, respectively.

A preprocessing step will substitute all blank nodes in G_1 for fresh blank nodes that do not occur in G_2. One can therefore assume from now on that G_1 and G_2 have no common blank nodes.

Since G_1 and G_2 are OWL 2 DL ontologies in RDF graph form, the *canonical parsing process* for computing the reverse OWL 2 RDF mapping, as described in Section 3 of [OWL 2 RDF Mapping], can be applied to map the graphs G_1 and G_2 to corresponding OWL 2 DL ontologies in Functional Syntax form. For the resulting ontologies it is then algorithmically possible to determine for every occurring IRI and anonymous individual all the entity types. By this, all missing declaration triples are added to G_1 and G_2.

Further, since G_1 and G_2 are OWL 2 DL ontologies in RDF graph form, the *canonical parsing process* can also be applied to safely identify all subgraphs of G_1 and G_2 that correspond to language constructs described in [OWL 2 Specification], including all the language constructs considered in the theorem.

Based on these observations, the following steps are performed on every subgraph $g \subseteq G_2$ that has been identified by the canonical parsing process:

- (a) If *g* is the RDF encoding of an *annotation*, then *g* will be removed from G_2.
- (b) If *g* is a *deprecation triple*, then *g* will be removed from G_2.
- (c) If *g* is an *ontology header* (which may include ontology properties, such as import directives), then *g* is substituted in G_2 by a triple of the form "*x* `rdf:type owl:Ontology`", where *x* is a fresh blank node. At the end of the process, all but one of these triples will be removed from G_2.
- (d) If *g* is the RDF encoding of an *entity declaration*, then *g* is copied to G_1.
- (e) If *g* is the RDF encoding of a *property expression* with root blank node *x*, then *g* together with the RDF triple "*x* `owl:equivalentProperty` *x*" is added to G_1.
- (f) If *g* is the RDF encoding of a *class expression* with root blank node *x*, then *g* together with the RDF triple "*x* `owl:equivalentClass` *x*" is added to G_1.

- (g) If *g* is the RDF encoding of a *data range expression* with root blank node *x*, then:
  - if *g* is part of a *data property restriction expression*, then nothing needs to be done, since this case is covered by the treatment of class expressions in (f);
  - if *g* is part of a *datatype definition*, then G_2 contains a triple "*u* owl:equivalentClass *x*" with some IRI *u* that is declared as a datatype, and then the triple and *g* are copied to G_1;
  - otherwise, *g* is part of at least one *data property range axiom*. One such property *p* is chosen randomly, and the RDF encoding *r* of a *universal property restriction expression* on property *p* is created for the datarange expression rooted by node *x*. Let the RDF graph *r* have fresh root blank node *y*. *r* together with the RDF triple "*y* `owl:equivalentClass` *y*" is added to G_1.

In the following it is shown that all the claims of the theorem hold.

A: Existence of a terminating algorithm. An algorithm exists for mapping the input pair $\langle G_1^*, G_2^* \rangle$ to the output pair $\langle G_1, G_2 \rangle$, since the canonical parsing process for the determination of the missing entity declarations and for the identification of the language construct subgraphs is described in the form of an algorithm in [*OWL 2 RDF Mapping*]. All other operations described above can obviously be performed algorithmically. The algorithm *terminates*, since the canonical parsing process terminates (including termination on invalid input), and since all other operations described above are executed by a finite number of steps, respectively.

B: The resulting RDF graphs are OWL 2 DL ontologies. Since the original RDF graphs G_1^* and G_2^* are OWL 2 DL ontologies in RDF graph form, this is also the case for G_1 and G_2, since each of the steps above transforms a pair of OWL 2 DL ontologies in RDF graph form again into a pair of OWL 2 DL ontologies in RDF graph form, for the following reasons:

- The substitution of existing blank nodes by fresh blank nodes does not change the structure of an OWL 2 DL ontology in RDF graph form.
- If an entity has some entity type but the corresponding entity declaration is omitted, then it may be added without harm.
- Annotations and deprecation statements can always be removed from an OWL 2 DL ontology, as done by (a) and (b), since they are non-required language constructs.
- Any OWL 2 DL ontology requires the existence of an ontology header, but does not require the existence of an ontology IRI, so it is sufficient to replace an ontology header by (c).
- G_1 contains entity declarations for every IRI occurring in G_1, since by (d) all entity declarations from G_2^* have been copied to G_1, and only IRIs from both G_1^* and G_2^* can occur in G_1 via (e), (f) and (g). It is not a problem to have entity declarations for IRIs that are not further used in an OWL 2 DL ontology.
- Adding a datatype definition to an OWL 2 DL ontology, as done by (g), will lead to a new OWL 2 DL ontology (note that the corresponding datatype declaration from G_2^* has been copied to G_1 as well).

- Adding syntactically valid OWL 2 DL axioms to an OWL 2 DL ontology, as done by (e), (f) and (g), will lead to a new OWL 2 DL ontology.

C: The resulting pair of RDF graphs is balanced. Property (1) of the theorem requires that the pair $\langle G_1 , G_2 \rangle$ is *balanced*. The following list checks that all the properties of the definition are satisfied.

- Property (1): F(G_1) and F(G_2) mutually meet the restrictions on OWL 2 DL ontologies, since the restrictions are mutually met by the original ontologies, and since only declarations and expressions that already existed in G_2^* are added to G_1 by (d), (e), (f) and (g). The removal of annotations by (a) and deprecation triples by (b) from G_2^*, as well as the replacement of the ontology header of G_2^* by (c) do not hurt any syntactic restrictions either.
- Property (2): All missing entity declaration triples have been added to G_1 and G_2.
- Properties (3) and (4): G_2 does not contain any RDF encodings of annotations nor deprecation triples due to their removal by (a) and (b), respectively.
- Property (5): G_2 contains exactly one ontology header consisting of a single triple of the form "*b* `rdf:type owl:Ontology`", where *b* is a blank node, due to the replacement of the existing ontology headers by (c).
- Property (6): Each RDF encoding of an entity declaration in G_1 also occurs in G_2, due to their copying by (d).
- Properties (7), (8) and (9): Each RDF encoding of a property, class or data range expression in G_1 also occurs in G_2, due to their adding by (e), (f) and (g), respectively.

D: The resulting ontologies are semantically equivalent with the original ontologies. Property (2) of the theorem requires that F(G_1) is semantically equivalent with F(G_1^*). This is the case, since F(G_1) differs from F(G_1^*) only by

- additional entity declarations (due to the addition of all missing entity declarations and due to (d)), which have no formal meaning under the OWL 2 Direct Semantics;
- datatype definitions (due to (g)), which only introduce a new name for a data range expression;
- additional OWL 2 DL axioms (due to (e), (f) and (g)), which are all, by construction, tautologies under the OWL 2 Direct Semantics.

Further, property (3) of the theorem requires that F(G_2) is semantically equivalent with F(G_2^*). This is the case, since F(G_2) differs from F(G_2^*) only by additional entity declarations, and missing annotations including deprecation annotations (due to (a) and (b)), which all have no formal meaning under the OWL 2 Direct Semantics.

End of the Proof of the Balancing Lemma.

In the following, the correspondence theorem will be proven.

Assume that the premises of the correspondence theorem hold for given RDF graphs G_1^* and G_2^*. This allows for applying the *balancing lemma*, which provides the existence of certain RDF graphs G_1 and G_2 that are OWL 2 DL ontologies in RDF graph form. Hence, it is possible to build OWL 2 DL ontologies in Functional Syntax form F(G_1) and F(G_2) by applying the reverse OWL 2 RDF mapping to G_1 and G_2, respectively.

The *balancing lemma* further provides that the pair $\langle G_1 , G_2 \rangle$ is *balanced*.

The claimed property (1) of the correspondence theorem follows directly from property (1) of the *balancing lemma* and from property (1) of the "Balanced"-definition. The claimed properties (2) and (3) of the correspondence theorem follow directly from properties (2) and (3) of the *balancing lemma*, respectively.

The rest of this proof will treat the claimed property (4) of the correspondence theorem, which states that if F(G_1) OWL 2 Direct entails F(G_2) with respect to *F(D)*, then G_1 OWL 2 RDF-Based entails G_2 with respect to *D*.

Let *I* be an OWL 2 RDF-Based interpretation w.r.t. an OWL 2 RDF-Based datatype map *D* of a vocabulary V_I that covers all the names (IRIs and literals) occurring in the RDF graphs G_1 and G_2, and let *I* OWL 2 RDF-Based satisfy G_1. It will be shown that *I* OWL 2 RDF-Based satisfies G_2.

As a first step, an OWL 2 Direct interpretation *J* w.r.t. the corresponding OWL 2 Direct datatype map *F(D)* will be constructed for a vocabulary V_J that covers all the names (IRIs and literals) occurring in the OWL 2 DL ontologies in Functional Syntax form F(G_1) and F(G_2). *J* will be defined in a way such that it closely corresponds to *I* on those parts of the vocabularies V_I and V_J that cover G_1 and G_2, and F(G_1) and F(G_2), respectively.

G_1 and G_2 are OWL 2 DL ontologies in RDF graph form that are mapped by the reverse RDF mapping to F(G_1) and F(G_2), respectively. This means that the same literals are used in both G_1 and F(G_1), and in both G_2 and F(G_2), respectively. Further, since the pair $\langle G_1 , G_2 \rangle$ is *balanced*, according to property (2) of the "Balanced"-definition there are entity declarations in F(G_1) and F(G_2) for all the entity types of every non-built-in IRI occurring in G_1 and G_2, respectively. For each entity declaration of the form `Declaration`(*T*(*u*)) in F(G_1) and F(G_2), where *T* is the entity type for some IRI *u*, a typing triple of the form *u* `rdf:type` *t* exists in G_1 or G_2, respectively, where *t* denotes the class representing the part of the universe that corresponds to *T*; and vice versa.

Since the pair $\langle G_1 , G_2 \rangle$ is *balanced*, all the entity declarations of F(G_2) are also contained in F(G_1), and therefore all the typing triples of G_2 that correspond to some entity declaration in F(G_2) are also contained in G_1. Since *I* OWL 2 RDF-Based satisfies G_1, all these "declaring" typing triples are OWL 2 RDF-Based satisfied by *I*, and thus all non-built-in IRIs in G_1 and G_2 are actually instances of their declared parts of the universe.

Based on these observations, the OWL 2 Direct interpretation J and its vocabulary V_J for the datatype map $F(D)$ can now be defined.

The vocabulary $V_J := (V_C , V_{OP} , V_{DP} , V_I , V_{DT} , V_{LT} , V_{FA})$ is defined as follows.

- The set V_C of class names contains all IRIs that are declared as classes in $F(G_1)$.
- The set V_{OP} of object property names contains all IRIs that are declared as object properties in $F(G_1)$.
- The set V_{DP} of data property names contains all IRIs that are declared as data properties in $F(G_1)$.
- The set V_I of individual names contains all IRIs that are declared as named individuals in $F(G_1)$.

The sets V_{DT} of datatype names, V_{LT} of literals, and V_{FA} of facet-literal pairs are defined according to Section 2.1 of [*OWL 2 Direct Semantics*] w.r.t. the datatype map $F(D)$. Specifically, V_{DT} includes all IRIs that are declared as datatypes in $F(G_1)$.

The interpretation $J := (\Delta_I , \Delta_D , \cdot^C , \cdot^{OP} , \cdot^{DP} , \cdot^I , \cdot^{DT} , \cdot^{LT} , \cdot^{FA})$ is defined as follows. The object and data domains of J are identified with the universe IR and the set of data values LV of I, respectively, i.e., Δ_I := IR and Δ_D := LV. The datatype interpretation function $\cdot^{DT}$, the literal interpretation function $\cdot^{LT}$, and the facet interpretation function $\cdot^{FA}$ are defined according to Section 2.2 of [*OWL 2 Direct Semantics*]. Specifically, $\cdot^{DT}$ interprets all IRIs that are declared as datatypes in $F(G_1)$ according the following definition. For every non-built-in IRI u occurring in $F(G_1)$:

- If u is declared as a named individual, then set $u^I := I(u)$, since the graph contains the triple "u `rdf:type owl:NamedIndividual`", i.e., $I(u) \in$ IR.
- If u is declared as a class, then set $u^C :=$ ICEXT($I(u)$), since the graph contains the triple "u `rdf:type owl:Class`", i.e., $I(u) \in$ IC.
- If u is declared as a datatype, then set $u^{DT} :=$ ICEXT($I(u)$), since the graph contains the triple "u `rdf:type rdfs:Datatype`", i.e., $I(u) \in$ IDC.
- If u is declared as an object property, then set $u^{OP} :=$ IEXT($I(u)$), since the graph contains the triple "u `rdf:type owl:ObjectProperty`", i.e., $I(u) \in$ IP.
- If u is declared as a data property, then set $u^{DP} :=$ IEXT($I(u)$), since the graph contains the triple "u `rdf:type owl:DatatypeProperty`", i.e., $I(u) \in$ IODP.

Note that G_1 may also contain declarations of annotation properties, but they will not be interpreted by the OWL 2 Direct Semantics and are therefore ignored here. This will not lead to problems, since the pair $\langle G_1 , G_2 \rangle$ is *balanced*, and therefore G_2 does not contain any annotations.

Further, note that the definition of *J* is compatible with the concept of a *non-separated vocabulary* in OWL 2 DL (also called *"punning"*, see Section 5.9 of [*OWL 2 Specification*]). Since G_1 and G_2 are OWL 2 DL ontologies in RDF graph form, it is allowed that the same IRI *u* is declared to be all of an individual name, and a class name, and either an object property name or a data property name. According to *I*, the IRI *u* will always denote the same individual in the universe IR, where $I(u)$ will be both a class and a property. Under *J*, however, the individual name *u* will denote an individual, the class name *u* will denote a subset of Δ_I, and the property name *u* will denote a subset of $\Delta_I \times \Delta_I$.

Literals occurring in G_1 and G_2 are mapped by the OWL 2 RDF mapping to the same literals in the corresponding interpreted language constructs of $F(G_1)$ and $F(G_2)$, which comprise data enumerations, has-value restrictions with a data value, cardinality restrictions, datatype restrictions, data property assertions, and negative data property assertions. Also, the semantics of literals is strictly analog for both the OWL 2 RDF-Based Semantics and the OWL 2 Direct Semantics. Therefore, literals need no further treatment in this proof.

Based on the premise that *I* OWL 2 RDF-Based satisfies G_1, it has to be shown that *J* OWL 2 Direct satisfies $F(G_1)$. For this to hold it will be sufficient to show that *J* OWL 2 Direct satisfies every axiom occurring in $F(G_1)$. Let *A* be an axiom occurring in $F(G_1)$, and let g_A be the subgraph of G_1 that is mapped to *A* by the reverse OWL 2 RDF mapping. It is possible to prove that *J* OWL 2 Direct satisfies *A* by showing that the meaning, which is given to *A* by the OWL 2 Direct Semantics, is compatible with the semantic relationship that, according to *J*, holds between the denotations of the names occurring in *A*. The basic idea is as follows:

Since *I* OWL 2 RDF-Based satisfies G_1, all the triples occurring in g_A are OWL 2 RDF-Based satisfied by *I*. Also, since *I* is an OWL 2 RDF-Based interpretation, all the OWL 2 RDF-Based semantic conditions are met by *I*. Hence, the left-to-right directions of all the semantic conditions that are "matched" by the triples in g_A will apply. This will reveal certain semantic relationships that, according to *I*, hold between the denotations of the names occurring in g_A. These semantic relationships are, roughly speaking, the semantic consequences of the axiom that is encoded by the triples in g_A.

Since the denotations w.r.t. *J* of all the names occurring in *A* have been defined in terms of the denotations and class and property extensions w.r.t. *I* of the same names occurring in g_A, and since the meaning of the axiom *A* w.r.t. the OWL 2 Direct Semantics turns out to be fully covered by the semantic consequences of the subgraph g_A w.r.t. the OWL 2 RDF-Based Semantics, one can eventually show that *J* OWL 2 Direct satisfies *A*.

A special note is necessary for *anonymous individuals* occurring in an assertion *A*. These have the form of the same blank node *b* both in *A* and in g_A. Both the OWL 2 RDF-Based Semantics and the OWL 2 Direct Semantics treat blank nodes as *existential variables* in an ontology. Since *I* satisfies g_A, *b* can be mapped to an individual *x* in IR such that g_A becomes true under *I* (see Section 1.5 in [*RDF Semantics*] for the precise definition on how blank nodes are treated in RDF based

languages). The same mapping from *b* to *x* can also be used for *J* in order to OWL 2 Direct satisfy *A*.

This basic idea is now demonstrated in more detail for a single example axiom *A* in F(G_1), which can be taken as a hint on how a complete proof could be constructed in principle. A complete proof would need to take every language construct of OWL 2 into account, as well as additional aspects such as datatype maps and facets. As in the example below, such a proof can make use of the observation that the definitions of the OWL 2 Direct Semantics and the OWL 2 RDF-Based Semantics are actually closely aligned for all the different language constructs of OWL 2.

Example:

Let *A* = `SubClassOf(ex:c1 ObjectUnionOf(ex:c2 ex:c3))` for IRIs `ex:c1`, `ex:c2` and `ex:c3` that are declared to be classes elsewhere in F(G_1).

Due to the reverse OWL 2 RDF mapping, g_A has the form

g_A :

```
ex:c1 rdfs:subClassOf _:x .
_:x rdf:type owl:Class .
_:x owl:unionOf ( ex:c2 ex:c3 ) .
```

Since *I* is an OWL 2 RDF-Based interpretation, it meets all the OWL 2 RDF-Based semantic conditions. Since *I* OWL 2 RDF-Based satisfies G_1, all the triples in g_A are OWL 2 RDF-Based satisfied, and this triggers the left-to-right directions of the semantic conditions for subclass axioms (`rdfs:subClassOf`) and union class expressions (`owl:unionOf`). This reveals that the denotations of the names in g_A are actually classes

I(`ex:c1`) ∈ IC ,
I(`ex:c2`) ∈ IC ,
I(`ex:c3`) ∈ IC ,

and that the following semantic relationship holds between the extensions of these classes:

ICEXT(I(`ex:c1`)) ⊆ ICEXT(I(`ex:c2`)) ∪ ICEXT(I(`ex:c3`)) .

From applying the definition of *J* follows that the following semantic relationship, w.r.t. *J*, holds between the denotations of the class names occurring in *A*:

(`ex:c1`) ^{C} ⊆ (`ex:c2`) ^{C} ∪ (`ex:c3`) ^{C} .

This semantic relationship equals the meaning of the axiom *A* = `SubClassOf(ex:c1 ObjectUnionOf(ex:c2 ex:c3))` w.r.t. the OWL 2 Direct Semantics. Hence, *J* OWL 2 Direct satisfies *A*.

Since *J* OWL 2 Direct satisfies F(G_1), and since F(G_1) OWL 2 Direct entails F(G_2), it follows that *J* OWL 2 Direct satisfies F(G_2).

The next step will be to show that *I* OWL 2 RDF-Based satisfies G_2. For this to hold, *I* needs to OWL 2 RDF-Based satisfy all the triples occurring in G_2, taking into account the premise that an OWL 2 RDF-Based interpretation is required to meet all the OWL 2 RDF-Based semantic conditions.

Since the pair $\langle G_1 , G_2 \rangle$ is *balanced*, G_2 contains a single ontology header consisting of a single triple "*b* `rdf:type owl:Ontology`" with a blank node *b*, and it does neither contain annotations nor deprecation statements. Hence, F(G_2) only consists of entity declarations and axioms, and does not have any ontology IRI and no ontology version, annotations or import directives. Further, since G_2 is an OWL 2 DL ontology in RDF graph form, every triple occurring in G_2, which is not the ontology header triple, belongs to some subgraph of G_2 that is mapped by the reverse OWL 2 RDF mapping to one of the entity declarations or axioms contained in F(G_2).

For the *ontology header* triple "*b* `rdf:type owl:Ontology`" in G_2: Since G_1 is an OWL 2 DL ontology in RDF graph form, G_1 contains an ontology header containing a triple "*x* `rdf:type owl:Ontology`", where *x* is either an IRI or a blank node. Since *I* OWL 2 RDF-Based satisfies G_1, this particular triple is satisfied by *I*. From the semantic conditions of *"Simple Entailment"*, as defined in [*RDF Semantics*], follows that the triple "*b* `rdf:type owl:Ontology`" with the existentially interpreted blank node *b* is satisfied by *I*, too.

For *entity declarations*, let *A* be an entity declaration in F(G_2), and let g_A *be the corresponding subgraph of* G_2. Since the pair $\langle G_1 , G_2 \rangle$ is *balanced*, *A* occurs in F(G_1), and hence g_A is a subgraph of G_1. Since *I* OWL 2 RDF-Based satisfies G_1, *I* in particular OWL 2 RDF-Based satisfies g_A.

For *axioms*, let *A* be an axiom in F(G_2), and let g_A be the corresponding subgraph of G_2. It is possible to prove that *I* OWL 2 RDF-Based satisfies g_A, by showing that all the premises for the right-to-left hand side of the particular semantic conditions, which are associated with the sort of axiom represented by g_A, are met. This will allow to apply the semantic condition, from which will follow that all the triples in g_A are OWL 2 RDF-Based satisfied by *I*. The premises of the semantic condition generally require that the denotations of all the non-built-in names in g_A are contained in the appropriate part of the universe, and that the semantic relationship that is expressed on the right hand side of the semantic condition actually holds between the denotations of all these names w.r.t. *I*. Special care has to be taken regarding the blank nodes occurring in g_A. The basic idea is as follows:

For every non-built-in IRI *u* occurring in g_A, *u* also occurs in *A*. Since the pair $\langle G_1 , G_2 \rangle$ is *balanced*, property (2) of the "Balanced"-definition provides that there are entity declarations in F(G_2) for all the entity types of *u*, each being of the form *E* := "`Declaration`(*T*(*u*))" for some entity type *T*. From the reverse RDF mapping follows that for each such declaration *E* a typing triple *e* exists in G_2, being of the form *e* := "*u* `rdf:type` *t*", where *t* is the name of a class representing the part of the universe corresponding to the entity type *T*. It has already been shown that for *E* being an entity declaration in F(G_2), and *e* being the corresponding subgraph in G_2, *I* OWL 2 RDF-Based satisfies *e*. Hence, *I*(*u*) is an individual contained in the appropriate part of the universe.

Further, since *J* OWL 2 Direct satisfies F(G_2), *J* OWL 2 Direct satisfies *A*. Therefore, the semantic relationship that is represented by *A* according to the OWL 2 Direct Semantics actually holds between the denotations of the names occurring in *A* w.r.t. *J*. Since the denotations of these names w.r.t. *J* have been defined in terms of the denotations and class and property extensions w.r.t. *I* of the same names in G_2, by applying the definition of *J* it will turn out that the analog relationship also holds between the denotations of the same names occurring in g_A.

Finally, for the blank nodes occurring in g_A, it becomes clear from the fact that G_2 is an OWL 2 DL ontology in RDF graph form that only certain kinds of subgraphs of g_A can occur having blank nodes.

- *Case 1:* A blank node corresponds to some anonymous individual in *A* (for *A* being one of a class assertion, object property assertion or data property assertion, according to Sections 5.6, 9.5 and 11.2 in [*OWL 2 Specification*]). The same blank node is used in *A*, and *J* interprets it as an existential variable. This renders the semantic relationship that is expressed by *A* into an existential assertion. After applying the definition of *J*, the analog existential assertion holds w.r.t. *I*, with the same blank node as the same existential variable in g_A.
- *Case 2:* A blank node is the "root" node of the multi-triple RDF encoding g_A of *A* (for g_A being an n-ary disjointness axiom from Section 5.10, or a negative property assertion from Section 5.15). The right-to-left direction of the semantic condition for this kind of axiom is of a form that the triples in g_A containing the blank node will be OWL 2 RDF-Based satisfied after all the premises of the semantic condition are met.
- *Case 3:* A blank node is the "root" node of the multi-triple RDF encoding g_E of an expression in *A* (for g_E being either a sequence, or one of a boolean connective from Section 5.4, an enumeration from Section 5.5, a property restriction from Section 5.6, or a datatype restriction from Section 5.7). Since the pair $\langle G_1 , G_2 \rangle$ is *balanced*, g_E also occurs in G_1. Since *I* OWL 2 RDF-Based satisfies G_1, g_E is OWL 2 RDF-Based satisfied, either, taking into account that blank nodes are existential variables. Hence, the left-to-right direction of the respective semantic condition for the particular sort of expression can be applied. This can be done for all the expressions occurring in g_A, and g_A can be seen as a directed acyclic graph with the triples encoding the actual axiom on top, and the different component

expressions being connected via blank nodes. Eventually, one can see that all the premises of the right-to-left direction of the semantic condition for the axiom encoded by g_A hold.

This basic idea is now demonstrated in more detail for a single example axiom A in F(G_2), which can be taken as a hint on how a complete proof could be constructed in principle. A complete proof would need to take every language construct of OWL 2 into account, as well as additional aspects such as datatype maps and facets. As in the example below, such a proof can make use of the observation that the definitions of the OWL 2 Direct Semantics and the OWL 2 RDF-Based Semantics are actually closely aligned for all the different language constructs of OWL 2.

Example:

Let A = `SubClassOf(ex:c1 ObjectUnionOf(ex:c2 ex:c3))` for IRIs `ex:c1`, `ex:c2` and `ex:c3` that are declared to be classes elsewhere in F(G_2).

Due to the reverse OWL 2 RDF mapping, g_A has the form

g_A :

```
ex:c1 rdfs:subClassOf _:x .
_:x rdf:type owl:Class .
_:x owl:unionOf ( ex:c2 ex:c3 ) .
```

The entity declarations for the class names `ex:c1`, `ex:c2` and `ex:c3` occurring in both A and g_A correspond to the typing triples

```
ex:c1 rdf:type owl:Class .
ex:c2 rdf:type owl:Class .
ex:c3 rdf:type owl:Class .
```

in G_2, respectively. Based on the premise that the pair $\langle G_1 , G_2 \rangle$ is *balanced*, all these typing triples are OWL 2 RDF-Based satisfied by I. Hence, all of the IRIs denote classes:

I(`ex:c1`) $\in$ IC ,
I(`ex:c2`) $\in$ IC and
I(`ex:c3`) $\in$ IC .

Since J OWL 2 Direct satisfies A, the following semantic relationship holds between the denotations of the class names in A w.r.t. J:

$(\texttt{ex:c1})^{C} \subseteq (\texttt{ex:c2})^{C} \cup (\texttt{ex:c3})^{C}$.

Applying the definition of J results in the following semantic relationship w.r.t. I that holds between the denotations of the names in g_A:

ICEXT(*I*(ex:c1)) ⊆ ICEXT(*I*(ex:c2)) ∪ ICEXT(*I*(ex:c3)) .

The subgraph g_E of g_A, given by

g_E :

```
_:x rdf:type owl:Class .
_:x owl:unionOf ( c2 c3 ) .
```

corresponds to a union class expression in *A*. Since the pair ⟨ G_1 , G_2 ⟩ is *balanced*, g_E is also a subgraph of G_1 (it will be assumed that the same blank nodes are used in both instances of g_E in order to simplify the argument). Since both G_1 and G_2 are OWL 2 DL ontologies in RDF graph form, the blank nodes occurring in g_E do not occur outside of g_E, neither in G_1 nor in G_2.

Since *I* OWL 2 RDF-Based satisfies G_1, according to the semantic conditions for RDF graphs with blank nodes (see Section 1.5 of [*RDF Semantics*]), a mapping *B* from blank(g_E) to IR exists, where blank(g_E) is the set of all blank nodes in g_E, such that the extended interpretation *I*+*B* OWL 2 RDF-Based satisfies all the triples in g_E. An analog argument holds for all the blank nodes occurring in the sequence expression `( c2 c3 )`.

This allows to apply the left-to-right direction of the semantic condition for union class expressions (`owl:unionOf`), providing:

[*I*+*B*](_:x) ∈ IC ,
ICEXT([*I*+*B*](_:x)) = ICEXT(*I*(ex:c2)) ∪ ICEXT(*I*(ex:c3)) .

Together with the intermediate results from above, it follows:

I(ex:c1) ∈ IC ,
[*I*+*B*](_:x) ∈ IC ,
ICEXT(*I*(ex:c1)) ⊆ ICEXT([*I*+*B*](_:x)) .

Therefore, all the premises are met to apply the right-to-left direction of the semantic condition of subclass axioms (`rdfs:subClassOf`), which results in

⟨ *I*(ex:c1) , [*I*+*B*](_:x) ⟩ ∈ IEXT(*I*(rdfs:subClassOf)) .

So, the triple

```
ex:c1 rdfs:subClassOf _:x .
```

is OWL 2 RDF-Based satisfied by *I*+*B*, where "_:x" is the same blank node as the root blank node of the union class expression in g_A.

Hence, w.r.t. existential blank node semantics, *I* OWL 2 RDF-Based satisfies all the triples in g_A.

To conclude, for every OWL 2 RDF-Based interpretation *I* that OWL 2 RDF-Based satisfies G_1 it turns out that *I* also OWL 2 RDF-Based satisfies G_2. Hence, G_1 OWL 2 RDF-Based entails G_2.

Q.E.D.

8 Appendix: Comprehension Conditions (Informative)

The correspondence theorem in Section 7.2 shows that it is possible for the OWL 2 RDF-Based Semantics to reflect all the entailments of the OWL 2 Direct Semantics [*OWL 2 Direct Semantics*], provided that one allows for certain "harmless" syntactic transformations on the RDF graphs being considered. This makes numerous potentially desirable and useful entailments available that would otherwise be outside the scope of the OWL 2 RDF-Based Semantics, for the technical reasons discussed in Section 7.1. It seems natural to ask for similar entailments even when an entailment query does not consist of OWL 2 DL ontologies in RDF graph form. However, the correspondence theorem does not apply to such cases, and thus the OWL 2 Direct Semantics cannot be taken as a reference frame for "desirable" and "useful" entailments, or for when a graph transformation can be considered "harmless" or not.

As discussed in Section 7.1, a core obstacle for the correspondence theorem to hold were RDF encodings of OWL 2 expressions, such as union class expressions, when they appear on the right hand side of an entailment query. Under the OWL 2 RDF-Based Semantics, it is not generally ensured that an individual exists, which represents the denotation of such an expression. The *"comprehension conditions"* defined in this section are additional semantic conditions that provide the necessary individuals for *every* sequence, class and property expression. By this, the combination of the normative semantic conditions of the OWL 2 RDF-Based Semantics (Section 5) and the comprehension conditions can be regarded to "simulate" the semantic expressivity of the OWL 2 Direct Semantics on entailment queries consisting of *arbitrary* RDF graphs.

The combined semantics is, however, not primarily intended for use in actual implementations. The comprehension conditions add significantly to the complexity and expressivity of the basic semantics and, in fact, have proven to lead to formal inconsistency. But the combined semantics can still be seen as a generalized reference frame for "desirable" and "useful" entailments, and this can be used, for example, to evaluate methods that syntactically transform *unrestricted* entailment queries in order to receive additional entailments under the OWL 2 RDF-Based Semantics. Such a concrete method is, however, outside the scope of this specification.

Note: The conventions in the introduction of Section 5 ("Semantic Conditions") apply to the current section as well.

8.1 Comprehension Conditions for Sequences

Table 8.1 lists the comprehension conditions for sequences, i.e. RDF lists. These comprehension conditions provide the existence of sequences built from any finite combination of individuals contained in the universe.

Table 8.1: Comprehension Conditions for Sequences

if	then exists z_1 , … , z_n ∈ IR
a_1 , … , a_n ∈ IR	⟨ z_1 , a_1 ⟩ ∈ IEXT(*I*(`rdf:first`)) , ⟨ z_1 , z_2 ⟩ ∈ IEXT(*I*(`rdf:rest`)) , … , ⟨ z_n , a_n ⟩ ∈ IEXT(*I*(`rdf:first`)) , ⟨ z_n , *I*(`rdf:nil`) ⟩ ∈ IEXT(*I*(`rdf:rest`))

8.2 Comprehension Conditions for Boolean Connectives

Table 8.2 lists the comprehension conditions for boolean connectives (see Section 5.4 for the corresponding semantic conditions). These comprehension conditions provide the existence of complement classes for any class, and of intersections and unions built from any finite set of classes contained in the universe.

Table 8.2: Comprehension Conditions for Boolean Connectives

if	then exists *z* ∈ IR
s sequence of c_1 , … , c_n ∈ IC	⟨ *z* , *s* ⟩ ∈ IEXT(*I*(`owl:intersectionOf`))
s sequence of c_1 , … , c_n ∈ IC	⟨ *z* , *s* ⟩ ∈ IEXT(*I*(`owl:unionOf`))
c ∈ IC	⟨ *z* , *c* ⟩ ∈ IEXT(*I*(`owl:complementOf`))
d ∈ IDC	⟨ *z* , *d* ⟩ ∈ IEXT(*I*(`owl:datatypeComplementOf`))

8.3 Comprehension Conditions for Enumerations

Table 8.3 lists the comprehension conditions for enumerations (see Section 5.5 for the corresponding semantic conditions). These comprehension conditions provide the existence of enumeration classes built from any finite set of individuals contained in the universe.

Table 8.3: Comprehension Conditions for Enumerations

if	then exists $z \in$ IR
s sequence of a_1 , … , $a_n \in$ IR	$\langle z , s \rangle \in$ IEXT(I(`owl:oneOf`))

8.4 Comprehension Conditions for Property Restrictions

Table 8.4 lists the comprehension conditions for property restrictions (see Section 5.6 for the corresponding semantic conditions). These comprehension conditions provide the existence of cardinality restrictions on any property and for any non-negative integer, as well as value restrictions on any property and on any class contained in the universe.

Note that the comprehension conditions for self restrictions constrains the right hand side of the produced `owl:hasSelf` assertions to be the boolean value `"true"^^xsd:boolean`. This is in accordance with Table 13 in Section 3.2.4 of [*OWL 2 RDF Mapping*].

Implementations are *not* required to support the comprehension conditions for `owl:onProperties`, but *may* support them in order to realize *n-ary dataranges* with arity ≥ 2 (see Section 7 of [*OWL 2 Specification*] for further information).

Table 8.4: Comprehension Conditions for Property Restrictions

if	then exists $z \in$ IR
$c \in$ IC , $p \in$ IP	$\langle z , c \rangle \in$ IEXT(I(`owl:someValuesFrom`)) , $\langle z , p \rangle \in$ IEXT(I(`owl:onProperty`))
$c \in$ IC , s sequence of p_1 , … , $p_n \in$ IP , $n \geq 1$	$\langle z , c \rangle \in$ IEXT(I(`owl:someValuesFrom`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:onProperties`))
$c \in$ IC , $p \in$ IP	$\langle z , c \rangle \in$ IEXT(I(`owl:allValuesFrom`)) , $\langle z , p \rangle \in$ IEXT(I(`owl:onProperty`))
$c \in$ IC , s sequence of p_1 , … , $p_n \in$ IP , $n \geq 1$	$\langle z , c \rangle \in$ IEXT(I(`owl:allValuesFrom`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:onProperties`))
$a \in$ IR , $p \in$ IP	$\langle z , a \rangle \in$ IEXT(I(`owl:hasValue`)) , $\langle z , p \rangle \in$ IEXT(I(`owl:onProperty`))
$p \in$ IP	$\langle z , I$(`"true"^^xsd:boolean`)$\rangle \in$ IEXT(I(`owl:hasSelf`)) , $\langle z , p \rangle \in$ IEXT(I(`owl:onProperty`))

$n \in$ INNI , $p \in$ IP	$\langle z , n \rangle \in$ IEXT(I(owl:minCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))
$n \in$ INNI , $p \in$ IP	$\langle z , n \rangle \in$ IEXT(I(owl:maxCardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))
$n \in$ INNI , $p \in$ IP	$\langle z , n \rangle \in$ IEXT(I(owl:cardinality)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))
$n \in$ INNI , $c \in$ IC , $p \in$ IP	$\langle z , n \rangle \in$ IEXT(I(owl:minQualifiedCardinality)) , $\langle z , c \rangle \in$ IEXT(I(owl:onClass)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))
$n \in$ INNI , $d \in$ IDC , $p \in$ IODP	$\langle z , n \rangle \in$ IEXT(I(owl:minQualifiedCardinality)) , $\langle z , d \rangle \in$ IEXT(I(owl:onDataRange)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))
$n \in$ INNI , $c \in$ IC , $p \in$ IP	$\langle z , n \rangle \in$ IEXT(I(owl:maxQualifiedCardinality)) , $\langle z , c \rangle \in$ IEXT(I(owl:onClass)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))
$n \in$ INNI , $d \in$ IDC , $p \in$ IODP	$\langle z , n \rangle \in$ IEXT(I(owl:maxQualifiedCardinality)) , $\langle z , d \rangle \in$ IEXT(I(owl:onDataRange)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))
$n \in$ INNI , $c \in$ IC , $p \in$ IP	$\langle z , n \rangle \in$ IEXT(I(owl:qualifiedCardinality)) , $\langle z , c \rangle \in$ IEXT(I(owl:onClass)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))
$n \in$ INNI , $d \in$ IDC , $p \in$ IODP	$\langle z , n \rangle \in$ IEXT(I(owl:qualifiedCardinality)) , $\langle z , d \rangle \in$ IEXT(I(owl:onDataRange)) , $\langle z , p \rangle \in$ IEXT(I(owl:onProperty))

8.5 Comprehension Conditions for Datatype Restrictions

Table 8.5 lists the comprehension conditions for datatype restrictions (see Section 5.7 for the corresponding semantic conditions). These comprehension conditions provide the existence of datatypes built from restricting any datatype contained in the universe by any finite set of facet-value pairs contained in the facet space (see Section 4.1) of the original datatype.

The set IFS is defined in Section 5.7.

Table 8.5: Comprehension Conditions for Datatype Restrictions

if	then exists $z \in$ IR , s sequence of z_1 , … , z_n $\in$ IR
$d \in$ IDC , f_1 , … , $f_n \in$ IODP , v_1 , … , $v_n \in$ LV , $\langle f_1 , v_1 \rangle$, … , $\langle f_n , v_n \rangle \in$ IFS(d)	$\langle z , d \rangle \in$ IEXT(I(`owl:onDatatype`)) , $\langle z , s \rangle \in$ IEXT(I(`owl:withRestrictions`)) , $\langle z_1 , v_1 \rangle \in$ IEXT(f_1) , … , $\langle z_n , v_n \rangle \in$ IEXT(f_n)

8.6 Comprehension Conditions for Inverse Properties

Table 8.6 lists the comprehension conditions for inverse property expressions. These comprehension conditions provide the existence of an inverse property for any property contained in the universe.

Inverse property expressions can be used to build axioms with anonymous inverse properties, such as in the graph

```
_:x owl:inverseOf ex:p .
_:x rdfs:subPropertyOf owl:topObjectProperty .
```

Note that, to some extent, the OWL 2 RDF-Based Semantics already covers the use of inverse property expressions by means of the semantic conditions of inverse property axioms (see Section 5.12), since these semantic conditions also apply to an existential variable on the left hand side of an inverse property axiom. Nevertheless, not all relevant cases are covered by this semantic condition. For example, one might expect the above example graph to be generally true. However, the normative semantic conditions do not permit this conclusion, since it is not ensured that for every property p there is an individual in the universe that happens to be the inverse property of p.

Table 8.6: Comprehension Conditions for Inverse Properties

if	then exists $z \in$ IR
$p \in$ IP	$\langle z , p \rangle \in$ IEXT(I(`owl:inverseOf`))

9 Appendix: Changes from OWL 1 (Informative)

This section lists relevant differences between the OWL 2 RDF-Based Semantics and the original specification of the *OWL 1 RDF-Compatible Semantics* [OWL 1

RDF-Compatible Semantics]. Significant effort has been spent in keeping the design of the OWL 2 RDF-Based Semantics as close as possible to that of the OWL 1 RDF-Compatible Semantics. While this aim was achieved to a large degree, the OWL 2 RDF-Based Semantics actually deviates from its predecessor in several aspects, in most cases due to serious technical problems that would have arisen from a conservative semantic extension. Not listed are the new language constructs and the new datatypes of OWL 2.

The following markers are used:

- **[DEV]**: A deviation from the OWL 1 RDF-Compatible Semantics that formally breaks backwards compatibility.
- **[EXT]**: An extension to the OWL 1 RDF-Compatible Semantics that is backwards compatible.
- **[NOM]**: A change of the nomenclature compared to that being used in the OWL 1 RDF-Compatible Semantics.

Generalized Graph Syntax [EXT]: The OWL 2 RDF-Based Semantics allows RDF graphs to contain *IRIs* [*RFC 3987*] (see Section 2.1), whereas the OWL 1 RDF-Compatible Semantics was restricted to RDF graphs with *URIs* [*RFC 2396*]. This change is in accordance with the rest of the OWL 2 specification (see Section 2.4 of [*OWL 2 Specification*]). In addition, the OWL 2 RDF-Based Semantics is now explicitly allowed to be applied to RDF graphs containing *"generalized" RDF triples*, i.e. triples that can consist of IRIs, literals or blank nodes in all three positions (Section 2.1), although implementations are not required to support this. In contrast, the OWL 1 RDF-Compatible Semantics was restricted to RDF graphs conforming to the RDF Concepts specification [*RDF Concepts*]. These limitations of the OWL 1 RDF-Compatible Semantics were actually inherited from the RDF Semantics specification [*RDF Semantics*]. The relaxations are intended to warrant interoperability with existing and future technologies and tools. Both changes are compatible with OWL 1, since all RDF graphs that were legal under the OWL 1 RDF-Compatible Semantics are still legal under the OWL 2 RDF-Based Semantics.

Datatype Facets [EXT]: The basic definitions of a *datatype* and a *D-interpretation*, as defined by the RDF Semantics specification and as applied by the OWL 1 RDF-Compatible Semantics, have been extended to take *constraining facets* into account (see Section 4), in order to allow for *datatype restrictions* as specified in Section 5.7. This change is compatible with OWL 1, since Section 5.1 of the RDF Semantics specification explicitly allows for extending the minimal datatype definition provided there.

Correspondence Theorem and Comprehension Conditions [DEV]: The semantic conditions of the OWL 1 RDF-Compatible Semantics included a set of so called *"comprehension conditions"*, which allowed to show the original *"correspondence theorem"* stating that every entailment of OWL 1 DL was also an entailment of OWL 1 Full. The document at hand adds comprehension conditions for the new language constructs of OWL 2 (see Section 8). However, the comprehension conditions are *not* a normative aspect of the OWL 2 RDF-Based Semantics anymore. It has turned out that combining the comprehension

conditions with the normative set of semantic conditions in Section 5 would lead to formal inconsistency of the resulting semantics (Issue 119). In addition, it became clear that a correspondence theorem along the lines of the original theorem would not work for the relationship between the OWL 2 RDF-Based Semantics and the OWL 2 Direct Semantics [*OWL 2 Direct Semantics*], since it is not possible to "balance" the differences between the two semantics solely by means of additional semantic conditions (see Section 7.1). Consequently, the correspondence theorem of the OWL 2 RDF-Based Semantics (Section 7.2) follows an alternative approach that replaces the use of the comprehension conditions and can be seen as a technical refinement of an idea originally discussed by the WebOnt Working Group (email). This change is an *incompatible deviation* from OWL 1, since certain aspects of the originally normative definition of the semantics have been removed.

Flawed Semantics of Language Constructs with Argument Lists [DEV]: In the OWL 1 RDF-Compatible Semantics, the semantic conditions for unions, intersections and enumerations of classes were defined in a flawed form, which lead to formal inconsistency of the semantics (Issue 120; see also an unofficial problem description). The affected semantic conditions have been revised; see Section 5.4 and Section 5.5. This change is an *incompatible deviation* from OWL 1, since the semantics has formally been weakened in order to eliminate a source of inconsistency.

Incomplete Semantics of `owl:AllDifferent` [EXT]: The OWL 1 RDF-Compatible Semantics missed a certain semantic condition for axioms based on the vocabulary term "`owl:AllDifferent`" (see also an unofficial problem description). The missing semantic condition has been added to the OWL 2 RDF-Based Semantics (see Section 5.10). This change is compatible with OWL 1, since the semantics has been conservatively extended.

Aligned Semantics of `owl:DataRange` and `rdfs:Datatype` [EXT]: The class `owl:DataRange` has been made an *equivalent class* to `rdfs:Datatype` (see Section 5.2). The main purpose for this change was to allow for the deprecation of the term `owl:DataRange` in favor of `rdfs:Datatype`. This change is compatible with OWL 1 according to an analysis of the relationship between the two classes in the OWL 1 RDF-Compatible Semantics (email).

Non-Empty Data Value Enumerations [DEV]: The semantic condition for enumerations of data values in Section 5.5 is now restricted to *non-empty* sets of data values. This prevents the class `owl:Nothing` from unintentionally becoming an instance of the class `rdfs:Datatype`, as analyzed in (email). This restriction of the semantics is an *incompatible deviation* from OWL 1. Note, however, that it is still possible to define an empty enumeration of data values, as explained in Section 5.5.

Terminological Clarifications [NOM]: This document uses the term *"OWL 2 RDF-Based Semantics"* to refer to the specified semantics only. According to Section 2.1, the term *"OWL 2 Full"* refers to the whole language that is determined by the set of RDF graphs (also called *"OWL 2 Full ontologies"*) being interpreted using the

OWL 2 RDF-Based Semantics. OWL 1 has not been particularly clear on this distinction. Where the OWL 1 RDF-Compatible Semantics specification talked about *"OWL Full interpretations"*, *"OWL Full satisfaction"*, *"OWL Full consistency"* and *"OWL Full entailment"*, the OWL 2 RDF-Based Semantics Specification talks in Section 4 about *"OWL 2 RDF-Based interpretations"*, *"OWL 2 RDF-Based satisfaction"*, *"OWL 2 RDF-Based consistency"* and *"OWL 2 RDF-Based entailment"*, respectively, since these terms are primarily meant to be related to the semantics rather than the whole language.

Modified Abbreviations [NOM]: The names "R_I", "P_I", "C_I", "EXT_I", "$CEXT_I$", "S_I", "L_I" and "LV_I" have been replaced by the corresponding names defined in the RDF Semantics document [*RDF Semantics*], namely "IR", "IP", "IC", "IEXT", "ICEXT", "IS", "IL" and "LV", respectively. Furthermore, all uses of the IRI mapping "IS" have been replaced by the more general interpretation mapping "*I*", following the conventions in the RDF Semantics document. These changes are intended to support the use of the OWL 2 RDF-Based Semantics document as an incremental extension of the RDF Semantics document. Names for the "parts of the universe" that were exclusively used in the OWL 1 RDF-Compatible Semantics document, such as "IX" or "IODP", have not been changed. Other abbreviations, such as "IAD" for the class extension of `owl:AllDifferent`, have in general not been reused in the document at hand, but the explicit non-abbreviated form, such as "IEXT(*I*(`owl:AllDifferent`))", is used instead.

Deprecated Vocabulary Terms [NOM]: The following vocabulary terms have been deprecated as of OWL 2 by the Working Group, and *should not* be used in new ontologies anymore:

- `owl:DataRange` (per resolution of Issue 29)

10 Appendix: Post Last-Call Changes (Informative)

Changes from the Last Call Working Draft of 21 April 2009:

- Renamed annotation vocabulary terms "owl:subject", "owl:predicate" and "owl:object" to "owl:annotatedSource", "owl:annotatedProperty" and "owl:annotatedTarget", respectively (per WG resolution).
- Replaced the datatype "rdf:text" by "rdf:PlainLiteral" (per WG resolution).
- Replaced the facet "rdf:langPattern" by "rdf:langRange", following the same replacement in the original "rdf:PlainLiteral" specification.
- Changed range of property "owl:annotatedProperty" from IP to IR in order to avoid undesired semantic side effects from annotations. This was an oversight when the original semantic conditions for annotations of axioms and annotations were removed from the document.
- Explained the optional status of the semantic conditions concerned with the IRI "owl:onProperties", in accordance with the rest of the OWL 2 specification.
- The semantic conditions and comprehension conditions for the n-ary property restrictions have been changed to only cover property sequences

of length greater than 0, since the meaning of an expression with an empty property set is not clear.
- Shortened and clarified some section titles, moved the section on semantic conditions for sub property chains within Section 5, and aligned the entry order of all tables in Section 8 with those in Section 5.
- Several editorial clarifications, minor corrections and cosmetic changes.

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[OWL 2 RDF Mapping]
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